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Rupture of liquid film, placed over deep fluid, under action of thermal load

A.S. Ovcharova *

Lavrentyev Institute of Hydrodynamics Siberian Branch, Russian Academy of Sciences, 15, Acad. Lavrentyev Avenue, Novosibirsk 630090, Russia

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ABSTRACT

In this paper, we compare the Marangoni effect and the effect of the Prandtl number variation on features of the liquid film rupture under the action of a thermal load and show a principal difference between these two effects. Results of computational analysis allow us to make the following conclusions. When affecting on the free surface of the plane liquid film by thermal beam, in order to obtain the initial holes of the same size in films with various thermal physical properties, one should apply a thermal load of a certain well-defined type, more specifically, concentrated thermal load, which provides the predetermined temperature at least on one of the film boundaries. In this case the size of the holes will depend only on the width of a thermal beam applied to the film free surface. Paper presents the solutions of the model problems.

1. Introduction

It is known that if the thermal load is applied on the free surface of a thin liquid film, the film is deformed and it can rupture. The rupture of the film can be accompanied by the formation of a drop, whose size depends on the type of the thermal load applied on the film free surface. Droplet generation conditions during the film rupture were investigated in [1,2].

The rupture of the film located on another liquid was studied experimentally and numerically by Bratukhin et al. [3]. It was shown, that the initial deformation and subsequent rupture of the film can be caused by various factors, such as an addition of a small dose of surfactant to the free surface of the film, local heating or an action of a directed air stream.

The thermal physical properties of the film affect the features of its rupture caused by an action of the thermal beam. There are two governing parameters associated with the temperature, which is generated under the action of the thermal load: these are Marangoni and Prandtl numbers. Majority of publications are related to investigations of Marangoni effect and other effects in thin liquid layer lying on a solid substrate [4–6]. In this paper, we investigate features of a film rupture depending on the Marangoni and the Prandtl numbers in film located over a deep liquid. We consider the deformation processes and rupture of the film assuming that the deep liquid does not move and is not deformed. In order to describe these processes, a mathematical model based on the two-dimensional Navier–Stokes equations is exploited, where the film is considered as the thin layer of a viscous non-isothermal liquid. Special attention in this work is focused on the derivation of the boundary conditions on interfaces between film and gas, as well as film and deep liquid.

2. Mathematical model

A plane thin liquid film with the density ρ, kinematical viscosity ν and surface tension coefficient $\sigma(T)$ is placed over another deep liquid with the density, which exceeds the density of the film. The liquids are immiscible and they are limited by two solid planes $x = 0$ and $x = L$. We assume that $y = 0$ is the boundary between the film and deep liquid and it is not deformed (see Fig. 1). In this figure, $y = f(t,x)$ is the free surface of the film in selected coordinates; h_0 is the film thickness at initial time moment; $\boldsymbol{g}$ is the gravity vector. Concentrated thermal load effects the film free surface in the form of thermal beam with a width d. Under the action of this thermal beam, the film changes its shape and further ruptures. Assuming that the liquid is incompressible, and the density and the viscosity are constants, we describe the film motion and the heat transfer by the Navier–Stokes equations which are written in terms of ψ (stream function), ω (vorticity), and the heat transfer equation for θ (temperature) [7–9]:

* Tel.: +7 383 333 17 82; fax: +7 383 333 16 12.
E-mail address: ovcharova@hydro.nsc.ru

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Nomenclature

t	time
x, y	coordinates
u, v	longitudinal and transverse velocity components
v_s, v_n	tangent and normal velocities of points lying on film free surface $f(t,x)$
$\overline{D}$	stress tensor
P	pressure
$\vec{s}$, $\vec{n}$	tangent and normal vectors
d	width of thermal beam
h/h_0	dimensionless film thickness
h_0	length scale
$\mathrm{v}_0 = \nu/h_0$	velocity scale
$P_0 = \rho \mathrm{v}_0^2$	pressure scale
g_0	gravity scale

Dimensionless parameters

$\mathrm{Pr} = \nu/a$	Prandtl number
a	thermal diffusivity
$\mathrm{Ca} = \rho \mathrm{v}_0 \nu/\sigma_0$	capillary number
$\mathrm{Mn} = \sigma_T \delta T/\rho \mathrm{v}_0 \nu$	Marangoni number
$\mathrm{G} = g_0 h_0/\mathrm{v}_0^2$	Galileo number
$\mathrm{Cr} = \sigma_T \delta T/\sigma_0$	crispation number

Greek symbols

ψ	stream function
ω	vorticity
θ	temperature
ν	kinematic viscosity
ρ	density

$$\frac{\partial \omega}{\partial t} + \frac{\partial}{\partial x}\left(\omega \frac{\partial \psi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\omega \frac{\partial \psi}{\partial x}\right) = \Delta \omega, \tag{1}$$

$$\Delta \psi + \omega = 0, \tag{2}$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial}{\partial x}\left(\theta \frac{\partial \psi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\theta \frac{\partial \psi}{\partial x}\right) = \frac{1}{\mathrm{Pr}} \Delta \theta. \tag{3}$$

Length, velocity and pressure scales are h_0, $\mathrm{v}_0 = \nu/h_0$ and $P_0 = \rho \mathrm{v}_0^2$, respectively, so that the Reynolds number is Re = 1. Here, Pr is the Prandtl number, $\theta = (T - T_0)/\delta T$ (T_0 is the characteristic temperature, δT is the temperature drop). The stream function and the vorticity are defined by the relations:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y},$$

where u and v are longitudinal and transverse components of the velocity $\vec{v}$ in selected co-ordinates (see Fig. 1).

We assume that initial conditions are based on the fact that the film is plane and does not move; the temperature $\theta(0,x,y) = 0$ for whole domain.

2.1. Boundary conditions on the free gas-film surface $f(t,x)$

Let us determine the normal and tangent vectors to the free surface $y = f(t,x)$ in any point as

$$\vec{n} = \left\{\frac{-f_x}{\sqrt{1+f_x^2}}, \frac{1}{\sqrt{1+f_x^2}}\right\}, \quad \vec{s} = \left\{\frac{1}{\sqrt{1+f_x^2}}, \frac{f_x}{\sqrt{1+f_x^2}}\right\}$$

and assume that the surface tension coefficient $\sigma(T)$ is a linear function of the temperature

$$\sigma(T) = \sigma_0(1 - \sigma_T(T - T_0)), \quad \sigma_T = -\frac{1}{\sigma_0}\frac{d\sigma}{dT}\bigg|_{T=T_0},$$

where $\sigma_0 = \sigma(T_0)$, $\sigma_T > 0$. The boundary conditions at the free surface are specified in the form of kinematic and dynamic conditions [10,11]. Ignoring the evaporation (condensation) processes and

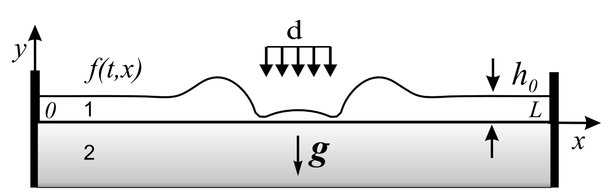

Fig. 1. Sketch showing the effect of the concentrated thermal load on the free surface of the film (1) lying on a deep liquid (2). Here, d is the width of heat beam, h_0 is the initial film thickness.

the effect of the dynamic characteristics of the gas on the motion of the liquid, the dynamic condition at the film-gas interface can be written as

$$\overline{D}\vec{n} = \frac{\sigma(T)}{R}\vec{n} + \nabla \sigma(T), \tag{4}$$

where R is the radius of the curvature of the free surface $f(t,x)$, ∇ is the surface tension gradient along the surface and $\overline{D}$ is the stress tensor:

$$\overline{D} = \begin{Bmatrix} -P + 2\rho\nu \frac{\partial u}{\partial x} & \rho\nu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \rho\nu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & -P + 2\rho\nu \frac{\partial v}{\partial y} \end{Bmatrix}.$$

In this work, we use the method based on the reduction of the solution to the boundary, where the balance relations (4) are specified. The implementation of this method is similar to the well-known procedure of reducing the solution to the boundary for an ideal fluid [12,13]. However, there are a number of significant differences related to the viscosity of the liquid.

Let us write kinematic condition

$$f_t = v - f_x u \tag{5}$$

and two consequences from the dynamic condition (4).

The condition of continuity of the tangent stress vector

$$\vec{s}(\overline{D})\vec{n} = \frac{\partial \sigma(T)}{\partial s}, \tag{6}$$

the condition of continuity of the normal stress vector

$$\vec{n}(\overline{D})\vec{n} = \frac{\sigma(T)}{R}. \tag{7}$$

Using kinematic condition (5) and Eq. (2), we can reduce the condition of continuity of the tangent stress (6) to the form

$$\omega = 2\left(\frac{v_s}{R} + \frac{\partial v_n}{\partial s}\right) + \frac{1}{\rho\nu}\frac{\partial \sigma(T)}{\partial s} = 2\left(\frac{v_s}{R} + \frac{\partial v_n}{\partial s}\right) + \frac{1}{\rho\nu}\frac{\partial \sigma(T)}{\partial T}\frac{\partial T}{\partial s},$$

which can be used both as the modified condition of continuity of the tangent stress and the formula for definition of the vorticity on the free surface.

The condition (7) can be transformed to the form

$$\frac{\sigma(T)}{R} = -P - 2\rho\nu\left(\frac{\partial v_s}{\partial s} - \frac{v_n}{R}\right). \tag{8}$$

Now, we project the Navier–Stokes vector equation written in the natural variables, i.e. the velocity $\vec{v}$ and the pressure P