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Original Research Paper

Understanding production of fines in batch ball milling for mill scale-up design using the population balance model

V.K. Gupta *

Department of Fuel and Mineral Engineering, Indian Institute of Technology (ISM), Dhanbad 826 004, India

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ABSTRACT

The rate of production of fine material in the batch mode of grinding operation forms the basis for determination of the grindability parameter of the Bond approach and the breakage distribution function of the population balance model (PBM) approach to the mill scale-up design. For a given set of mill operating conditions, the rate of production of fines is determined by the breakage characteristics and production history of the material being ground. Another important aspect is the variation in the rate of production of fines with grinding time. With a view to developing a clear understanding of these aspects, a detailed analysis of variations in the rate of production of fines was carried out using the PBM framework and two well-known functional forms for the specific breakage rate and breakage distribution parameters. In this paper, it has been shown how the results of this analysis can be used for: (i) obtaining more accurate estimates of the breakage distribution parameters by performing just one short-duration batch grinding experiment, and (ii) explaining variation in the Bond Work index with the product size in terms of the exponent of particle size in the expression for the specific breakage rate function: $k_j = A^* x_j^\alpha$.

1. Introduction

Production of a ground product of desired fineness with minimum energy consumption is the most challenging job of a comminution engineer. As the energy consumption per unit weight of the product is determined by the rate of production of material of desired fineness and power drawn by the mill [1], a good understanding of the factors that affect these two performance parameters is important. In this paper, we analyze and discuss the rate of production of fines in the ball milling operation.

There are two main approaches to the scale-up design of ball mills: the Bond approach [2–4] and the population balance model (PBM) approach [1,5–10]. In both the cases, the rate of production of fine material of the desired size is determined by carrying out some batch grinding experiments, which are specific to the approach used. The feed charge for these experiments is generally prepared by crushing the material in a jaw and/or roll crusher [1–13]. The strength and shape distributions of particles, which determine the breakage characteristics of particles, undergo a significant change when the crushed material is subjected to further size reduction in the ball mill environment [1,9–11]. The effect of these changes on the specific breakage rate of particles as well as the specific rate of production of fines has been found to be quite significant [1,11]. Thus, this aspect has relevance to the experiments conducted for determination of the specific breakage rate and breakage distribution parameters of the population balance model as well as the grindability parameter of the Bond model. Determination of the specific breakage rate parameters has already been discussed in detail [11]. For determination of the breakage distribution parameters, the methods proposed by Herbst and Fuerstenau [12] and Austin and Luckie [13] involve grinding the crushed material for a short time. In light of the statements made above, it can be seen that the breakage distribution function thus obtained is valid only for the particles produced by the crusher and not for those produced in the ball mill. In the Bond test also, after the first batch grinding experiment, the feed charge consists of the crushed material as well as the ground product of the previous experiment that is coarser than the classifying (closing) screen. Both the factors, the differences in the breakage characteristics of the same size particles belonging to the two components of the feed and variation in the proportion of the two components, contribute to variation in the value of grindability in the successive experiments.

Another important aspect is variation in the rate of production of fines with grinding time. It is well known that in a ball mill when a single size feed is ground in the batch mode of operation, fine

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* Address: B-402 Bansal Plaza, Station Road, Ranchi 834 001, India.
E-mail address: gupta.vinaig@gmail.com

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Nomenclature

A^*	constant in Eq. (19), min^{-1}
$b_{i,j}$	breakage distribution function in discrete size intervals, dimensionless
$B(x, v)$	cumulative breakage distribution function in continuous size, dimensionless
$B_{i,j}$	cumulative breakage distribution function in discrete size intervals, dimensionless
C_i	defined in Eq. (31), dimensionless
d	ball diameter, cm
D	mill diameter, cm
E	specific energy input to the mill, kWh/t
Er_i	percent deviation in the F_i value from zero-order production behaviour
F_i	mass fraction of particulate charge finer than x_i, dimensionless
G_i	grindability corresponding to the classifying screen aperture size of x_i, g/rev
J	fraction of the mill volume occupied by balls at rest, dimensionless
$k(v)$	specific breakage rate of particles of size v, min^{-1}
k_i	specific breakage rate for particles of in i-th size interval, min^{-1}
$M_i(t)$	mass fraction of particulate charge in i-th size interval at time t, dimensionless
M_j^*	mass fraction in the j-th size interval of the mill feed, dimensionless
N	mill speed expressed as fraction of critical speed, dimensionless
$P_{i,j}$	specific production rate of material finer than size x_i from size fraction j, min^{-1}
S	aperture size of the classifying screen, μm
t	grinding time, min
v	particle size, μm
V	a constant in Eq. (33c)
Wi	Work index, kWh/t
x	particle size, μm
x_i	size of the openings of the lower sieve of the i-th size interval, μm
x_p, x_f	80% passing sizes associated with the product and feed in Eq. (5), μm
X_p, X_f	80% passing sizes associated with the test sieve undersize in the last cycle of the Bond test and the new feed to the ball mill in Eq. (6), μm

Greek symbols

α	exponent in Eq. (19), dimensionless
β	exponent in Eq. (20), dimensionless
δ	ratio of apertures of upper and lower sieves of a size interval, dimensionless
θ	exponent in Eq. (20), dimensionless
ϕ^*	constant in Eq. (20), dimensionless

Subscripts

i, j, m, n	indices for particle size interval, #

particles are initially produced at a constant rate (known as the ‘zero-order production of fines’) [5,12,14–17]. As grinding progresses, depending on the material being ground, the rate of production of fines is found to either decrease or increase. However, what breakage characteristics lead to a particular type of behavior in respect of the change in the rate of production of fines with grinding time is presently not well understood. This aspect has relevance to the design of experiments for a direct determination of the breakage distribution function.

Further, different trends of variation in the Bond work index with the choice of the classifying (closing) screen are observed for different materials [18]. In most cases, the value of the work index is found to increase with a decrease in the size of the openings of the classifying screen, but in some cases it is found to remain practically constant or even decrease [18]. A proper explanation for these observations is presently not available in the literature.

In this paper, we present a detailed analysis of the fines production kinetics using the framework of the size-discrete population balance mathematical model and discuss how the results of this analysis can be used for carrying out the various tasks mentioned above with greater accuracy and efficiency, and understanding observed variations in the specific energy consumption with the material breakage characteristics and product size.

2. Background

2.1. The population balance model

In its size-discrete form, the linear, time invariant population balance mathematical model of the batch grinding operation can be expressed by the following equation [8,19–21]

$$\frac{dM_i(t)}{dt} = -k_i M_i(t) + \sum_{j=1}^{i-1} b_{i,j} k_j M_j(t) \qquad (1)$$

where i is an index for a size interval that is bounded by the size of the openings of the upper and lower sieves, x_{i-1} and x_i, respectively; $M_i(t)$ is the mass fraction of the particulate charge in the size interval i; k_j is the size-discrete specific breakage rate parameter which represents the fractional rate at which particles break out of the size interval j; and $b_{i,j}$ is the breakage distribution parameter, which represents the mass fraction of the particles breaking out of the size interval j that reports to a finer size interval i. It may be mentioned that according to the notation followed here, the size interval 1 contains the coarsest particles. An alternative form of the model equation is [1,12,20,21]

$$\frac{dF_i(t)}{dt} = \sum_{j=1}^{i} B_{i,j} k_j M_j(t) \qquad (2)$$

where $F_i(t)$ is the mass fraction of the particulate material finer than size x_i and $B_{i,j}$ is the cumulative breakage distribution parameter, which represents the mass fraction of the material breaking out of the size interval j that is finer than size x_i. Let the product term $B_{i,j}k_j$, which will repeatedly appear in our analysis, be denoted by $P_{i,j}$, that is

$$P_{i,j} = B_{i,j} k_j \qquad (3)$$

The physical meaning of $P_{i,j}$ can be expressed as the specific rate of production of material finer than size x_i from particles of size class j. In conformity with the terms ‘specific breakage rate parameters’ and ‘breakage distribution parameters’ for the k and B parameters, the P parameters can be appropriately called as the ‘specific production rate parameters’. It may be noted that by definition $B_{j,j} = 1$, therefore we have

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