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Stability of natural convection in a vertical couple stress fluid layer

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ABSTRACT

The stability of buoyancy-driven parallel shear flow of a couple stress fluid confined between vertical plates is investigated by performing a classical linear stability analysis. The plates are maintained at constant but different temperatures. A modified Orr–Sommerfeld equation is derived and solved numerically using the Galerkin method with wave speed as the eigenvalue. The critical Grashof number G_c, critical wave number a_c and critical wave speed c_c are computed for wide ranges of couple stress parameter Λ_c and the Prandtl number *Pr*. Based on these parameters, the stability characteristics of the system are discussed in detail. The value of Prandtl number, at which the transition from stationary to travelling-wave mode takes place, increases with increasing Λ_c. The couple stress parameter shows destabilising effect on the convective flow against stationary mode, while it exhibits a dual behaviour if the instability is via travelling-wave mode. The streamlines and isotherms presented demonstrate the development of complex dynamics at the critical state.

1. Introduction

Buoyancy driven flows continued to receive the attention of researchers in both heat transfer and fluid mechanics. In recent years important industrial problems in which natural convection dominates are of major focus for analysts and experimentalists. The study of fluid motions and transport processes by buoyancy have been generally motivated by the important applications such as nuclear reactor, cooling of electronic equipments, materials processing such as solidification phenomenon, atmospheric and oceanic circulations, or in air currents rising from a cooling object, crystal growth processes, and other natural convection processes in the natural calamity (spread of fire). Cooling of electronic components by natural convection is most preferable as it is highly reliable and avoids additional power consumption to induce the flow as in the case of forced convection. The recognition of high free convection heat transfer rates in atomic reactors, electrical transformers and other engineering applications prompted many to understand and study the stability of natural convection. The main interest in the study of stability of natural convection in a fluid layer is to know when and how laminar flow breaks down, its subsequent development and its eventual transition to turbulence.

The stability of natural convection of a Newtonian viscous fluid which is confined between two parallel vertical plates maintained at constant and different temperatures provides one of the simplest cases of an interaction between buoyancy and shearing forces and has been investigated analytically, numerically and experimentally [1–9]. Instability of the base flow in such a vertical fluid layer occurs when the Grashof number becomes greater than a certain critical value. The most interesting observation is that the type of instability is determined by the magnitude of the Prandtl number *Pr*. For values of $Pr < 12.7$, the parallel flow undergoes a transition to a stationary multicell flow pattern when the Grashof number exceeds a critical value. This transition has been observed experimentally by Vest and Arpaci [6]. The critical disturbance modes are found to be travelling waves when $Pr > 12.7$.

Majority of the studies on the stability of natural convection in a vertical fluid layer are mainly concerned with Newtonian fluids which have a linear relationship between the shear stress and shear rate. However, fluid dynamical systems encountered in many practical problems cited above exhibit non-Newtonian behaviour. Therefore, studying the stability of natural convection considering non-Newtonian effects are quite desirable. Unlike Newtonian fluids, there are different kinds of non-Newtonian fluids and obviously they do not lend themselves to a unified treatment. In recent years, polar fluids – a class of non-Newtonian fluids have received a wider attention. These fluids deform and produce a spin field due to the microrotation of suspended particles. As far as

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Nomenclature

a	vertical wave number
c	wave speed
c_r	phase velocity
c_i	growth rate
$D = d/dx_1$	differential operator
$\vec{g}$	acceleration due to gravity
$G = \alpha g\beta h^4/\nu^2$	Grashof number
h	thickness of the dielectric fluid layer
p	pressure
$Pr = \nu/\kappa$	Prandtl number
$\vec{q} = (u_1, u_2, u_3)$	velocity vector
t	time
T	temperature
T_1	temperature of the left boundary
T_2	temperature of the right boundary
W_b	basic velocity
(x_1, x_2, x_3)	Cartesian co-ordinates

Greek symbols

α	thermal expansion coefficient
η	couple stress viscosity
κ	thermal diffusivity
$\Lambda_c = h\sqrt{\mu/\eta}$	couple stress parameter
μ	fluid viscosity
$\nu(=\mu/\rho_0)$	kinematic viscosity
$\psi(x_1, x_3, t)$	stream function
Ψ	amplitude of vertical component of perturbed velocity
ρ	fluid density
ρ_0	reference density at T_0
θ	amplitude of perturbed temperature

these types of non-Newtonian fluids are concerned, there are two important theories proposed by Eringen [10] and Stokes [11] and these are, respectively, referred to as micropolar fluid theory and couple stress fluid theory. The micropolar fluids take care of local effects arising from microstructure and as well as the intrinsic motions of microfluidics. The spin field due to microrotation of freely suspended particles set up an anti-symmetric stress, known as couple stress, and thus forming couple stress fluid. The couple-stress fluid theory represents the simplest generalisation of the classical viscous fluid theory that allows for polar effects and whose microstructure is mechanically significant in fluids. Moreover, the couple stress fluid model is one of the numerous models that were proposed to describe response characteristics of non-Newtonian fluids. The constitutive equations in these fluid models can be very complex and involve a number of parameters, also the resulting flow equations lead to boundary value problems in which the order of differential equations is higher than the Navier–Stokes equations and are given by Stokes [11] which allows the sustenance of couple stresses in addition to usual stresses. This fluid theory shows all the important features and effects of couple stresses and results in equations that are similar to Navier–Stokes equations. Couple-stress fluids have applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plates in a bath, nuclear slurries, exotic lubricants and colloidal fluids, liquids containing long-chain molecules as polymeric suspensions, and lubrication, electro-rheological fluids to mention a few.

Work on the stability of natural convection in a vertical fluid layer subsequently extended to non-Newtonian fluids is concerned only with viscoelastic fluids ([12,13]). Jain and Stokes [14] studied the effect of couple stresses in fluids on the hydrodynamic stability of plane Poiseuille flow, while effect of couple stresses on thermal convective instability is analyzed by many researchers ([15–19]). Rudraiah et al. [20] investigated electrohydrodynamic stability of couple stress fluid flow in a horizontal channel occupied by a porous medium using energy method.

Nonetheless, the effect of couple stresses on the stability of natural convection in a vertical fluid layer has not received any attention in the literature despite its relevance and importance in many practical problems cited above. The intent of the present paper is to investigate this problem in which the vertical plates are maintained at constant but different temperatures. Modified Orr–Sommerfeld equations are derived and the resulting eigenvalue problem is solved numerically using the Galerkin method.

2. Mathematical formulation

The geometric arrangement of the problem is illustrated schematically in Fig. 1. We consider an incompressible couple stress fluid confined between two parallel vertical plates at $x_1 = \pm h$. The left surface is maintained at fixed temperature T_1, whereas the plate at $x_1 = h$ is maintained at fixed temperature T_2 ($>T_1$). A Cartesian coordinate system (x_1, x_2, x_3) is chosen with the origin in the middle of the vertical fluid layer, where the x_1-axis is taken perpendicular to the plates and the x_3-axis is vertically upwards, opposite in the direction to the gravity. Under the Oberbeck–Boussinesq approximation (since the temperature difference between the vertical plates is assumed to be small, the density is treated as a constant everywhere in the governing equation except in the gravitational term), we have

$$u_{i,i} = 0 \tag{1}$$

$$\rho = \rho_0\{1 - \alpha(T - T_0)\} \tag{2}$$

where u_i is the velocity vector, T is the temperature, ρ is the fluid density, α is the thermal expansion coefficient, ρ_0 is the density at reference temperature $T = T_0$ (at the middle of the channel).

The equation of motion for couple stress fluids are based on the constitutive equations which are given by Stokes [11]. The stress tensor τ_{ij} consists of symmetric and anti-symmetric parts and

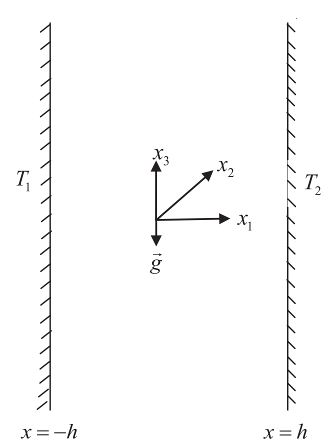

Fig. 1. Physical configuration.