International Journal of Heat and Mass Transfer 78 (2014) 670–687

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Numerical modeling of the non-isothermal liquid droplet impact on a hot solid substrate

Sashikumaar Ganesan [a,*], Sangeetha Rajasekaran [a], Lutz Tobiska [b]

[a] Numerical Mathematics and Scientific Computing, SERC, Indian Institute of Science, Bangalore 560 012, India
[b] Institute of Analysis and Numerical Computation, OvG University, PF4120, 39016 Magdeburg, Germany

ARTICLE INFO

Article history:
Received 11 November 2013
Received in revised form 12 June 2014
Accepted 8 July 2014
Available online 5 August 2014

Keywords:
Finite elements
ALE approach
Moving mesh
Impinging droplets
Moving contact line
Dynamic contact angle
Heat transfer

ABSTRACT

The heat transfer from a solid phase to an impinging non-isothermal liquid droplet is studied numerically. A new approach based on an arbitrary Lagrangian–Eulerian (ALE) finite element method for solving the incompressible Navier–Stokes equations in the liquid and the energy equation within the solid and the liquid is presented. The novelty of the method consists in using the ALE-formulation also in the solid phase to guarantee matching grids along the liquid–solid interface. Moreover, a new technique is developed to compute the heat flux without differentiating the numerical solution. The free surface and the liquid–solid interface of the droplet are represented by a moving mesh which can handle jumps in the material parameter and a temperature dependent surface tension. Further, the application of the Laplace–Beltrami operator technique for the curvature approximation allows a natural inclusion of the contact angle. Numerical simulation for varying Reynold, Weber, Peclét and Biot numbers are performed to demonstrate the capabilities of the new approach.

1. Introduction

The interaction between sprays and hot solid objects occur in a wide variety of industrial and environmental applications. Nevertheless, our understanding of the mechanisms involved in the process is far from being complete. Therefore, the study of the hydrodynamic and thermodynamic behavior of a single droplet impinging on a solid substrate is of fundamental importance.

As a droplet nears a hot solid substrate, heat is transferred from the solid to the liquid phase by conduction, convection and radiation. This energy is used to increase the temperature of the liquid or to vaporize the liquid from the base of the droplet. In the latter case a direct contact between the solid and the liquid phase is excluded (Leidenfrost phenomenon). For a surface temperature below the Leidenfrost temperature we suppose that the direct liquid-wall contact and the kinetic of the droplet spreading dominate the heat transfer. Thus, we consider in this paper the coupled heat transfer process in a single deforming droplet and in the solid phase during the spreading and recoiling from the moment of its impact till losing a direct contact with the solid. Despite several advances made in the field of Computational Fluid Dynamics, modeling and simulation of these processes are still very challenging. The Volume-of-Fluid [1–5], Level set [6–10], Immersed Boundary/Front Tracking [11–15] and the arbitrary Lagrangian Eulerian [16–21] are the most commonly used methods for tracking/capturing moving interfaces/boundaries.

Although several interface capturing/tracking methods have been proposed in the literature, only a few numerical studies incorporated the thermodynamic behavior. In the early study [22], MAC-type solution technique has been used for the computations of a water droplet impinging on a flat surface above the Leidenfrost temperature. However, the unsteady heat transfer computation has been neglected in this study. A considerable number of numerical simulations using the Volume-of-Fluid method have been reported in the literature for droplets impinging on a hot surface [23–29]. Also, numerical studies using the Level set method [30,31] and the Immersed Boundary method [32,33] have been reported in the literature. All these methods can be classified as fixed grid (Eulerian) methods. Computations using the Lagrangian approach for a liquid droplet impinging on a hot solid substrate have been presented in [34]. To the best of the authors knowledge, computations of a liquid droplet impinging on a hot solid substrate using the arbitrary Lagrangian–Eulerian (ALE) approach in both phases have not been reported in the literature.

In this paper, we present an accurate and efficient sharp interface ALE finite element approach for the computation of a non-isothermal liquid droplet impinging on a hot solid substrate. The Marangoni force, the surface force, and the jumps in the material

* Corresponding author.
E-mail addresses: sashi@serc.iisc.in (S. Ganesan), sangeetha@nmsc.serc.iisc.ernet.in (S. Rajasekaran), tobiska@ovgu.de (L. Tobiska).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.07.019
0017-9310/

parameters are incorporated into the model very accurately since the free surface and the liquid–solid interface are resolved by the computational mesh in the ALE approach. Also, spurious velocities arising often in other methods can be suppressed by using this approach [35]. Further, we use the Laplace–Beltrami operator technique [36,37] to treat the curvature in a semi-implicit manner [38]. This technique allows us to include the equilibrium contact angle into the model weakly [39]. Since the moving liquid–solid interface is resolved by the computational mesh, the energy equation in both the liquid and the solid phases can be solved by a one-field formulation. This allows us also to develop a new technique for computing the heat flux from the solid phase to the liquid phase without differentiating the numerical solution.

The paper is organized as follows. In Section 2 we introduce the governing equations for the fluid flow and the heat transfer in the liquid and solid phases. Then, the complex coupled problem is formulated in dimensionless quantities. Section 3 is devoted to the ingredients of our numerical approach. First, we explain in detail how the curvature approximation by the Laplace–Beltrami technique and the contact angle condition can be implemented in the weak form of the Navier–Stokes equations. We derive a one-field formulation for the temperature in the liquid–solid domain. Next the ALE approach to handle the moving mesh is discussed. Then, under the assumption of axisymmetry, the fully weak 3D formulation is transferred to the weak axisymmetric formulation. The discretization in space and time, in particular, the semi-discretization of the curvature term are given. Finally, we address the mesh handling techniques at the liquid–solid interface and the inner mesh update. Section 4 is concerned with numerical tests for varying Reynolds, Weber, Peclét and Biot numbers showing the capabilities of the new method. We shortly summarize the proposed numerical method and the obtained results in Section 5.

2. Mathematical model

2.1. Governing equations for the fluid flow

We consider a liquid droplet impinging on a horizontal hot substrate, see Fig. 1. The fluid is assumed to be incompressible; density and viscosity are constant. The governing equations for the fluid flow describing the sequence of spreading and recoiling of an impinging droplet in the given time interval (0, I) are the time-dependent incompressible Navier–Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \frac{1}{\rho}\nabla\cdot(\mathbb{T}(\mathbf{u},p)) = g\mathbf{e} \quad \text{in } \Omega_F(t)\times(0,\mathrm{I}), \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0 \quad \text{in } \Omega_F(t)\times(0,\mathrm{I}), \tag{2}$$

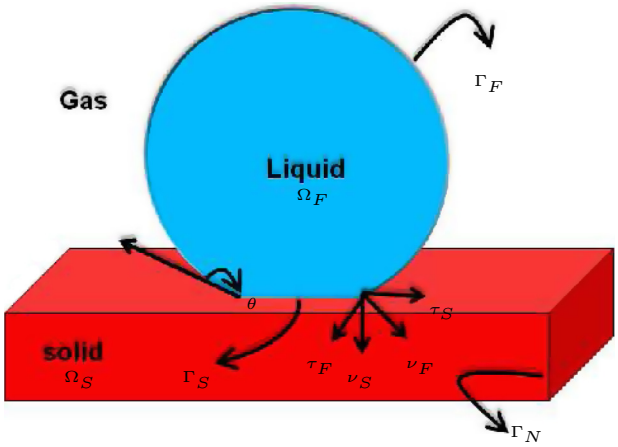

Fig. 1. Computational model for an impinging droplet on a hot solid substrate.

with the initial condition

$$\mathbf{u}(\cdot,0) = \mathbf{u}_0 \quad \text{in } \Omega_F(0) \tag{3}$$

and the boundary conditions given below. The time-dependent domain of the deforming droplet is denoted by $\Omega_F(t)$. The set $\Omega_F(t)\times(0,I)$ is understood as $\{(x,t)\in\mathbb{R}^4 : t\in(0,I), x\in\Omega_F(t)\}$. The boundary of the droplet satisfies $\partial\Omega_F(t) = \Gamma_F(t)\cup\Gamma_S(t)$, where $\Gamma_F(t)$ and $\Gamma_S(t)$ are the free surface and the liquid–solid interface of the droplet, respectively. Further, $\mathbf{u}$ is the velocity, p is the pressure, ρ is the density, g is the gravitational constant, t is the time, $\mathbf{e}$ is an unit vector in the opposite direction of the gravitational force, $\mathbf{u}_0(0,0,-u_{imp})$ is a given initial velocity and I is a given final time. The stress tensor $\mathbb{T}(\mathbf{u},p)$ for a Newtonian incompressible fluid is given by

$$\mathbb{T}(\mathbf{u},p) := 2\mu\mathbb{D}(\mathbf{u}) - p\mathbb{I}, \quad \mathbb{D}(\mathbf{u})_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right),$$

with $i,j = 1,\dots,3$, where μ denotes the dynamic viscosity, $\mathbb{D}(\mathbf{u})$ the velocity deformation tensor and $\mathbb{I}$ the identity tensor. The kinematic and force balancing conditions on the free surface Γ_F are given by

$$\mathbf{u}\cdot\boldsymbol{\nu}_F = \mathbf{w}\cdot\boldsymbol{\nu}_F \quad \text{on } \Gamma_F(t), \tag{4}$$

$$\mathbb{T}(\mathbf{u},p)\cdot\boldsymbol{\nu}_F = \sigma(T_F)\mathcal{K}\boldsymbol{\nu}_F + \underline{\nabla}\sigma(T_F) \quad \text{on } \Gamma_F(t). \tag{5}$$

Here, $\boldsymbol{\nu}_F$ denotes the unit outer normal vector on the free surface, $\mathcal{K}$ denotes the sum of the principal curvatures, $\underline{\nabla}$ the tangential gradient (defined in Section 3.1), $\mathbf{w}$ the velocity of the computational domain $\Omega_F(t), T_F$ the temperature on $\Gamma_F(t), \sigma(T_F)$ the temperature-dependent surface tension. Let $\boldsymbol{\tau}_{i,F}, i = 1,2$, denote tangential vectors on the free surface Γ_F(t). Then, (5) is equivalent to

$$\boldsymbol{\nu}_F\cdot\mathbb{T}(\mathbf{u},p)\cdot\boldsymbol{\nu}_F = \sigma(T_F)\mathcal{K}, \quad \text{on } \Gamma_F(t),$$
$$\boldsymbol{\tau}_{i,F}\cdot\mathbb{T}(\mathbf{u},p)\cdot\boldsymbol{\nu}_F = \boldsymbol{\tau}_{i,F}\cdot\nabla\sigma(T_F) \quad \text{on } \Gamma_F(t),$$

since the tangential gradient on $\Gamma_F(t)$ is perpendicular to $\boldsymbol{\nu}_F$. Further, for simplicity we assume that the temperature-dependent surface tension $\sigma(T_F)$ follows a simple linear law

$$\sigma(T_F) = \sigma_{ref} - \mathcal{C}_1(T_F - T_{ref}), \tag{6}$$

where T_{ref} is some reference temperature, σ_{ref} is the surface tension coefficient at the reference temperature and $\mathcal{C}_1 > 0$ is the negative rate of change of surface tension with temperature. The changes in the local surface temperature $T_F$on the free surface induce variations in the surface tension. Due to these variations in the surface tension, the fluid moves away from the region of low surface tension to the region of high surface tension. This effect is called Marangoni convection. Note that for large T_F the surface tension (6) could become negative and the model would be no longer correct. In order to avoid this unphysical behavior we check in our numerical computations whether the computed surface tension values are in the range of validity of the law (6).

Next, for moving contact line problems the conventional no-slip boundary condition at fixed walls cannot be used along the solid surface. Imposing the no-slip boundary condition at the moving contact line, where the free surface and the liquid–solid interface meet, leads to an non-integrable force singularity at the contact line [40–43]. Therefore, we use instead the Navier–slip boundary condition [39,44–46] on the liquid–solid interface $\Gamma_S(t)$

$$\mathbf{u}\cdot\boldsymbol{\nu}_S = 0, \quad \mathbf{u}\cdot\boldsymbol{\tau}_{i,S} + \epsilon_\mu(\boldsymbol{\tau}_{i,S}\cdot\mathbb{T}(\mathbf{u},p)\cdot\boldsymbol{\nu}_S) = 0. \tag{7}$$

Here, $\boldsymbol{\nu}_S$ and $\boldsymbol{\tau}_{i,S}, i = 1,2$ denote the outer unit normal and tangential vectors on the liquid–solid interface. Further, ϵ_μ is the slip coefficient [39,47].

2.2. Heat transfer from the solid into the liquid phase

The heat transfer in the moving droplet and in the fixed solid phase are described by the energy equation. In this study, we