Advanced Powder Technology xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Advanced Powder Technology

journal homepage: www.elsevier.com/locate/apt

Original Research Paper

Dynamic and perturbative system analysis of granular material in a vibrating screen

Nicolus Rotich[a,b,*], Ritva Tuunila[a], Ali Elkamel[b], Marjatta Louhi-Kultanen[a]

[a] Lappeenranta University of Technology (LUT), School of Engineering Sciences, P.O Box 20 FI-53851, Lappeenranta, South Karelia, Finland
[b] University of Waterloo, Chemical Engineering, 200 University Avenue West, Waterloo, ON N2L 3G1, Canada

ARTICLE INFO

Article history:
Received 4 May 2016
Received in revised form 14 February 2017
Accepted 27 September 2017
Available online xxxx

2015 MSC:
2017/02/14
15:18:40

Keywords:
Granular media
Error estimation
Dynamical systems
Nonlinear dynamics
Flow stability

ABSTRACT

In most particulate classification systems, feed rates in excess of 80% of the designed capacity leads to inefficiency and conversely feed rates below this value significantly diminishes the operational efficiencies. It therefore implies that maximum efficiency is only attainable at the expense of low capacity, and *vice versa*. This problem is caused by transience in granular flow due to start-ups and fluctuating feed-rates, in addition to fluctuations in feed material properties. If these variations are not checked, they cause instabilities, resulting in chaotic saddles responsible for in-process systemic error generation. These errors produce intermittent disruptions in production process and control. We have applied perturbation theory to study the effects of infinitesimal changes on the material balance analysis of the unit operation. The problem was identified as one of the highly *multi-stable* dynamic systems, characterized by 'predator-prey' phenomenon in dynamical systems theory. The study allowed formulation of optimal state equations, whose numerical solutions resulted in establishment of optimal operating conditions required to sustain stability, and consistently high tonnages and efficiency up to 99% simultaneously. The study also led to development of an optimization algorithm, which upon validation with experimental data showed a close relationship, with a minimal absolute error of 0.8 and a relative error of 6%. Finally, a representative case study was conducted on screen dimensioning, based on the determined parameters. Successful evolution of this methodology may be applied for up-scaling of real systems in future.

1. Introduction

Even though vibrating screens have been used in industries for over a century, it has often been cited as challenging in terms of understanding its nature of operation, particularly those that affect the dynamics of particles' physicochemical properties, and related design aspects [1–6]. Sometimes this ambiguity is created by complexity of the granular material properties themselves, e.g. surface phenomena [7]. Granules behave differently from the commonly known form of matter, to an extent some researchers propose should be considered a different state of matter on its own right [8]. The present work primarily focuses on the small variations caused by error generation and successive propagation in the granular flow and balance on vibrating screens. These problems when not checked, accumulates and thereby cause intermittent disruptions that may end up in an overall process economics turmoil.

Apart from a clean cut, higher efficiencies and capacities are the main objectives when evaluating vibrating screens' improvement[9,10]. A particular unique problem with vibrating and linear screens in general, is the antagonistic nature between the efficiency, η of particles' collection, and the feed rate capacity or tonnage (t/m^{2}h). This is one of the problems caused by transience in granular flow, such as start-ups, irregular feed-rates, and fluctuations in the feed properties, as explained by Spurling et *al.* in [11]. Fluctuations in the feed rates in particular (Fig. 1) has been known to cause detrimental chaotic effects in process control operations, e.g. causing premature cut-off or overshooting of delay time in dynamic response. [12–14]. In such occurrences, the transmission efficiency reduces if the screen must handle feed in excess of 80% of the rated tonnage and conversely, it also falls if the feed drops below this percentage [9]. Maximum efficiency is thus only attainable at the expense of low capacity and *vice versa*

---

* Corresponding author at: Lappeenranta University of Technology (LUT), School of Engineering Sciences, P.O. Box 20, FI-53851 Lappeenranta, South Karelia, Finland.
E-mail address: Nicolus.Rotich@Student.lut.fi (N. Rotich).

https://doi.org/10.1016/j.apt.2017.09.031

Please cite this article in press as: N. Rotich et al., Dynamic and perturbative system analysis of granular material in a vibrating screen, Advanced Powder Technology (2017), https://doi.org/10.1016/j.apt.2017.09.031

Nomenclature

a, b	α and β on MATLAB code	$\dot{m}_y$	underflow mass rate, kg/s
d_p	mean particles' diameter, m	$\dot{m}_x$	overflow mass rate, kg/s
F	feed, g	$u_{1\ldots4}$	masses of particles collected on 1, 2, 3 and4 mm screens, g
ϵ	first order perturbation parameter, dimensionless	U	experimental underflow after time t, g
f	objective function	t	runtime taken by batch mass, s
α	unperturbed constant related to increase in underflow	t_b	bed thickness, m
α_0	zeroth order perturbed undersized rate constant	SS	steady-state
α_1	first order perturbed undersized rate constant	v_y	underflow velocity, m/s
β	constant related to decrease in Feed	v_x	Overflow velocity, m/s
β_{max}	maximum β	V	material volume, m^3
ρ_B	bulk density of material, kg/m^3	w	screen width, m
ϕ	holed area fraction, dimensionless	X	component X in the particles, dimensionless
L	screen length, m	X_u	component X in the undersized particles, dimensionless
M_u	underflow mass, g	X_o	component X in the oversized particles, dimensionless
M_o	overflow mass, g	X_F	component X in the feed, dimensionless
δ	fractional uncertainty, dimensionless	η	Particles' separation efficiency, dimensionless
m_1	initial undersized mass, g		
m_i	elemental mass inflow, kg		
m_o	elemental mass outflow, kg		
m	instantaneous undersized mass, g		

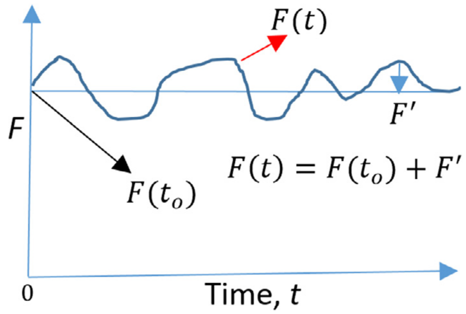

Fig. 1. An illustration of fluctuations in the feed rates.

[5,9,14–16]. King [9] particularly underscore the fact that older methods of performance evaluation concentrated entirely on capacity limitations, but more modern methods are beginning to consider the efficiency too in evaluating the overall performance of the plant. This is in agreement with Olsen and Carnes who in their technical paper stated that most screening applications do not require 100% size separation, and the lower this threshold is set, the higher the capacity, and this justifies the fact many vibrating screen manufactures today use target efficiencies of 90–95%, when not otherwise specified [16]. This is also seen in the efficiency model discussed by King [9].

While plant operators can make a choice on which to optimize between capacity and efficiency, different circumstances beforehand dictate necessary actions. For instance, in the gold mines, it might make economic sense to have an equipment that has a 50 t/m^2h capacity, and concentrates 0.9 g/g-ore, than having one with a massive 300 t/m^2h capacity, but an efficiency of 0.4 g/g-ore. On the contrary, this may not be true for a pulverized coal beneficiation plant with a biomass co-firing rate of 50/50 % (where the biomass share do not need any screening).

The second problem linked to the first one, is the existence of a 'secular equilibrium-like' state, in which the overflow reduces as the underflow is building up, both occurring at the same time, but at different rates, due to uncertainties of particulate dynamics. This 'predator-pray' interaction presents a challenge in estimating each rate inherently, and therefore both must be estimated simultaneously. This phenomenon is an example of a *multi-stable* dynamic system state. *Multi-stability* is the coexistence of more than one attractor for a given set of parameters. This phenomenon is found in almost all directions of research in natural sciences [17]. Material flow in vibrating screens particularly exhibit this relationship, in which the screen act as the 'predator', trapping the undersized particles representing the 'prey'. Some elementary but very useful notes on these relationships have been thoroughly explained by Bossel [18]. To get rid of these problems, most models make a silent assumption that these two rates are equal. This hypothesis actually represents the ideal, desired process at steady-state, and the rate constants are not precisely equal, but the actual amounts or particles' population. In reality however, these two rates vary significantly, owing to intermittent changes in the operating conditions, e.g. deck inclination, feed rate, screening time, aperture sizes, and other factors, related to the particles themselves e.g. size, shape, roughness etc., listed in [5]19.

Today, many designers use the modified version of the Vibrating Screen Manufactures Association (VSMA) to calculate screen capacities and efficiencies [16]. The VSMA formula takes 12 factors into consideration, all of which empirically obtained from industry experts. A recent reference has placed up to 22 factors which can affect the screening area design, even though not all of them have been put to use [20]. These factors are summarized into four categories: material, media, machine and duty. Fig. 2.

The present study is an attempt to address the above problems by studying the parameters that can be tweaked towards attaining a balanced material flow and loss recovery, for a vibrating screen. This is done by introducing infinitesimal changes (perturbations) onto hypothetical *best estimations* of the well known material balance equations of vibrating screens. The resulting system will then be solved numerically by applying *perturbation theory* models for error analyses discussed in the next sections. A successful insight to these uncertainties will further improve the understanding and development of better performing i.e. (optimal) linear screens. In particular, we try to answer the question, can we still achieve a reasonable efficiency, even with high tonnages? We also seek to establish system stability at various ranges of rate constants that allows optimal efficiencies at determined capacities. This is achieved through formulating and solving complex dynamics state equations related to material flows in a vibrating screen.

Please cite this article in press as: N. Rotich et al., Dynamic and perturbative system analysis of granular material in a vibrating screen, Advanced Powder Technology (2017), https://doi.org/10.1016/j.apt.2017.09.031