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A solution to the turbulent Graetz problem by matched asymptotic expansions for an axially rotating pipe subjected to external convection

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ABSTRACT

Heat transfer in an axially rotating pipe subjected to turbulent internal flow is strongly suppressed by tube rotation. For increasing rotational speeds of the pipe wall, the flow starts to laminarize and eventually the axial velocity profile of the hydrodynamically fully developed flow approaches the one of a laminar pipe flow. The solution for the heat transfer of a fluid with a hydrodynamically fully developed velocity profile in an axially rotating pipe subjected to external convection leads to a Sturm–Liouville eigenvalue problem. Asymptotic expressions are shown for the eigenvalues and constants based on a matched asymptotic expansion approach. These values are compared with numerical calculations and good agreement is found.

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1. Introduction

Fluid flow and heat transfer in rotating pipes are not only of considerable theoretical interest, but also of great practical importance. Technical applications are for example a rotating power transmission shaft which is longitudinally bored and through which cooling air in a gas turbine is guided or a rotating heat exchanger. There exists a large number of publications in literature which are concerned with the flow and heat transfer in axially rotating pipes where only some few [1–5] are cited here. For a laminar inlet flow into an axially rotating pipe it was found that the rotation destabilizes the flow and the tangential velocity profile $v_\varphi/v_{\varphi w}$ starts to deviate slightly from the linear distribution (a solid body rotation). In contrast, for a turbulent inlet flow into the pipe it has been observed in several publications [2,3] that the tangential velocity profile for the hydrodynamically fully developed turbulent flow can be assumed to be prescribed by $v_\varphi/v_{\varphi w} = (r/R)^2$. With this assumption the axial velocity profile can be predicted for the hydrodynamically fully developed flow by using a modified mixing length model with good agreement compared with experiments [2,3]. The heat transfer for thermal developing flow can then be predicted by analytically solving a Nusselt–Graetz problem. This has been done for the cases of constant wall temperature, constant wall heat flux and external convection [3–5] and good agreement with experimental data has been obtained. The problem with this kind of analytical solutions of the Nusselt–Graetz problem is in general that the sums appearing in this sort of solutions might require a large number of eigenvalues and constants for the here considered cases, because of the fact that the flow undergoes a laminarization from the turbulent to the laminar flow state with increasing rotation rates $N = \mathrm{Re}_\varphi/\mathrm{Re}_D$. This means also that the length of the thermal entrance region might increase to very large values for a flow with higher Reynolds numbers. The thermal entry length might then approach the one of a laminar flow, which is about $L_{th} \sim 0.05D\mathrm{Re}_D\mathrm{Pr}$. Thus, asymptotic expressions for the eigenvalues and constants are very useful for these cases. Expressions for asymptotic eigenvalues have been developed for Nusselt–Graetz problems for stationary tubes in the past by several authors by using the method of matched asymptotic expansions. Sleicher et al. [6] and Notter and Sleicher [7] developed asymptotic expressions for turbulent pipe flow through a non-rotating pipe for the two boundary conditions of a constant wall temperature and a constant wall heat flux. In doing so, the area between pipe center and pipe wall had to be subdivided into three regions. Analytical solutions for the related eigenvalue problem have then been obtained for each of the regions and have then been matched. The velocity profile was not defined in their works by an analytic function, but the asymptotic expressions contain the gradient of the velocity distribution at the pipe wall. For a non-rotating tube with external convection asymptotic expressions have been obtained by Sadeghipour et al. [8]. However, in their expressions the velocity profile has been explicitly described for a non-rotating pipe with a fully developed turbulent velocity profile as a power law representation. Thus, these

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Nomenclature

a	thermal diffusivity = $k/(\rho c_p)$ [m^2/s]
c_p	specific heat at constant pressure [J/(kg K)]
D	pipe diameter [m]
h	heat transfer coefficient inside the pipe [W/(m^2 K)]
h_a	external heat transfer coefficient [W/(m^2 K)]
N	rotation rate = $\mathrm{Re}_\varphi/\mathrm{Re}_D$ [–]
R	pipe radius [m]
k	thermal conductivity [W/(m K)]
k	modified eigenvalue [–]
Nu_D	Nusselt number based on pipe diameter = hD/k [–]
Pr	Prandtl number = ν/a [–]
Pr_t	turbulent Prandtl number = $\varepsilon_h/\varepsilon_m$ [–]
Re_D	Reynolds number based on D = $\bar{v}_z D/\nu$ [–]
Re_φ	rotational Reynolds number = $v_{\varphi w}D/\nu$ [–]
T	temperature [K]
T_0	constant inlet temperature [K]
T_b	bulk-temperature [K]
T_a	constant ambient temperature [K]
v_z, v_φ	axial and tangential velocity component [m/s]
$\bar{v}_z$	axial mean velocity [m/s]
$v_{\varphi w}$	tangential velocity of the pipe wall [m/s]
z, r, φ	axial, radial and tangential coordinates [m]

Greek symbols

ε_h	eddy diffusivity [m^2/s]
ε_m	eddy kinematic viscosity [m^2/s]
ρ	density [kg/m^3]
Δ	external convection parameter = $\chi R/k$ [–]
λ_j	eigenvalue [–]
Θ	dimensionless temperature [–]
ν	kinematic viscosity [m^2/s]
ϑ	modified eigenfunction [–]
χ	overall heat transfer coefficient [W/(m^2 K)]
Φ_j	eigenfunction [–]

expressions cannot be used for the current application where the flow field is strongly varying with rotation rate.

The aim of the present paper is twofold. First we want to develop asymptotic expressions for the case of external convection for an axially rotating pipe. These expressions will be based on the excellent analysis of Sleicher et al. [6] and Notter and Sleicher [7]. The second aim of the paper is to show a rigorous comparison between eigenvalues, constants and Nusselt numbers predicted by matched asymptotic expansions and values from numerical solutions of the underlying Sturm–Liouville eigenvalue problem. Such comparisons have only been rarely shown before (see for example Weigand [9] for the case of constant wall temperature and constant wall heat flux) and show nicely the accuracy and usefulness of the matched asymptotic expansions for these complicated cases with external convection. Of course, such comparisons have been hardly possible at the time the asymptotic expressions in [6,7] have been developed. As a result of the analysis Nusselt number expressions are given here which can easily been calculated by using the first three numerically predicted eigenvalues and constants together with the approximations presented here.

2. Analysis

Fig. 1 shows the geometrical configuration and the used coordinate system. The turbulent flow is assumed to be incompressible and hydrodynamically fully developed. The fluid properties are assumed to be constant. The axial velocity profiles for the hydrodynamically fully developed flow can be easily calculated (see [2,3]) and are shown in Fig. 2 together with some experimental results [5]. It is clearly visible how the turbulent flow gets laminarized with increasing rotation rate N and how the axial velocity profiles approach with increasing N the one of a laminar pipe flow [3]. Due to the constant fluid properties the equations of motion and the energy equation are decoupled. The temperature distribution and the heat transfer can be calculated by solving the energy equation for this case. Here we assume that the pipe is cooled externally by a constant ambient temperature T_a and a constant external heat transfer coefficient h_a at the outside of the rotating pipe. The pipe has an outer radius R_a an inner radius R, the pipe wall has the constant thermal conductivity k_w. This leads then to the overall heat transfer coefficient χ, defined by

$$\frac{1}{\chi} = \frac{R}{h_a R_a} + \frac{R}{k_w} \ln\left(\frac{R}{R_a}\right) \qquad (1)$$

The flow is assumed to be hydrodynamically fully developed and to enter the heated pipe section at $z = 0$ with a constant temperature T_0. The energy equation is given in dimensionless form by

$$\tilde{r}\tilde{v}_z(\tilde{r})\frac{\partial\Theta}{\partial\tilde{z}} = \frac{\partial}{\partial\tilde{r}}\left[E(\tilde{r})\tilde{r}\frac{\partial\Theta}{\partial\tilde{r}}\right] \qquad (2)$$

with the boundary conditions

$$\begin{aligned} &\tilde{z} = 0: \quad \Theta = 1 - T_a/T_0 \\ &\tilde{z} > 0, \quad \tilde{r} = 0: \quad \partial\Theta/\partial\tilde{r} = 0 \\ &\tilde{z} > 0, \quad \tilde{r} = 1: \quad \partial\Theta/\partial\tilde{r} + \Delta\Theta = 0 \end{aligned} \qquad (3)$$

and the dimensionless quantities

$$\begin{aligned} &\Theta = \frac{T - T_a}{T_0}, \quad \tilde{z} = \frac{z}{D}\frac{4}{\mathrm{Re}_D\mathrm{Pr}}, \quad \tilde{r} = \frac{r}{R}, \quad \tilde{v}_z = \frac{v_z}{\bar{v}_z}, \quad \mathrm{N} = \frac{v_{\varphi w}}{\bar{v}_z}, \\ &Nu = \frac{hD}{k}, \quad \mathrm{Re}_D = \frac{\bar{v}_z D}{\nu}, \quad \mathrm{Re}_\varphi = \frac{v_{\varphi w}D}{\nu}, \quad \mathrm{Pr} = \frac{\nu}{a}, \quad \Delta = \frac{\chi R}{k} \end{aligned} \qquad (4)$$

The function $E(\tilde{r})$ contains the eddy diffusivity of the turbulent flow and has been modeled according to [2,3] by using a modified mixing length approach:

$$E(\tilde{r}) = 1 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t}\frac{\varepsilon_m}{\nu} = 1 + \frac{\mathrm{Re}_D\mathrm{Pr}}{2\mathrm{Pr}_t}\left(\frac{l}{R}\right)^2\left[\left(\frac{\partial\tilde{v}_z}{\partial\tilde{r}}\right)^2 + (\tilde{r}N)^2\right]^{1/2} \qquad (5)$$

The quantity Δ in the boundary condition given by Eq. (3) is a modified Biot number. For $\Delta \to 0$ the case of an adiabatic wall is obtained, whereas for $\Delta \to \infty$ a constant wall temperature is achieved. Predictions with this method are in very good agreement

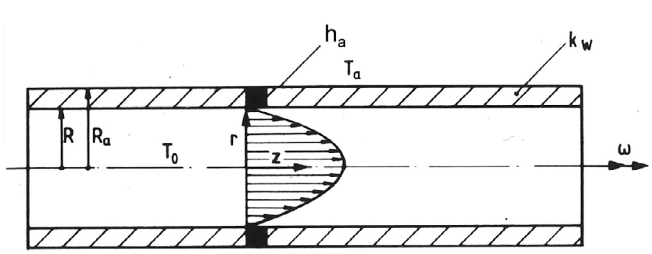

Fig. 1. Geometrical configuration and coordinate system (figure taken from [5]).

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