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The effect of rotation on radial horizontal convection and Nusselt number scaling in a cylindrical container

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ABSTRACT

The effect of rotation on horizontal convection flow in a free-surface cylindrical enclosure driven by a radially increasing temperature profile along the base is investigated numerically and theoretically. The governing equations of mass, momentum and energy subject to Boussinesq approximation applied to gravity term, have been discretised using a spectral-element method for velocity and temperature fields. Results of a scaling analysis are compared with numerical simulations at a fixed Prandtl number $Pr = 6.14$, Reynolds numbers up to 3200, and Rayleigh number up to 3.2×10^{11} in an enclosure with height-to-radius ratio $H/R = 0.4$. The results show that heat transfer in rotating horizontal convection is significantly affected by rotation, and where rotation effects are significant, Nusselt number scalings adapted from Park and Whitehead and Stern describe the behaviour at moderate and high rotation rates, respectively. A scaling analysis is conducted to describe the suppression of convective flow at high rotation rates. Flows are characterised in terms of a rotation parameter and are divided into three regimes: a diffusive regime with Nusselt number independent of thermal forcing and rotation, a rotation-affected convective regime, and a convective regime unaffected by rotation at sufficiently high Rayleigh number.

1. Introduction

Horizontal convection defines flows that are driven by temperature differentials imposed along a horizontal boundary [3]. This is in contrast to Rayleigh–Bénard convection in which the temperature differential is in the vertical direction [4,5]. Uneven heating applied across a horizontal boundary occurs in myriad geophysical and industrial systems, motivating further study into horizontal convection. In addition, the effect of rotation on convection flows is important in many industrial applications as well as in astrophysical and geophysical flows, including meridional overturning circulation in the ocean [6], Earth's core [7], as well as solar and mantle convection [8,9]. The combination of a radially forced horizontal convection and rotation in a cylindrical system idealises features of geophysical flows such as polar vortices in which solar heating of the surface has a latitudinal dependence, and this forms the basis of the system considered in the present study.

Laboratory fluid models for the study of polar vortices (e.g. [10,11]) have tended to feature mechanical forcing mechanisms such as differentially rotating disks that are dissimilar to the geophysical processes, though these models have typically featured mechanical forcing mechanisms dissimilar to atmospheric mechanisms. In the present paper, the flow in a rotating cylindrical container driven by horizontal convection with radial forcing is considered. Cylinder rotation mimics Earth's rotation, and radial horizontal convection drives an annular fluid flux circulating outward radially near the base and returning poleward at the top surface. Conservation of angular momentum accelerates the angular velocity of poleward-moving fluid, spinning it into a vortex in an analogous manner to the generation of atmospheric polar vortices in the polar convection cell.

As a first approximation of the atmospheric system, this model disregards beta-plane effects associated with the change in the Coriolis effect with latitude in the vicinity of the pole [10]. Moreover, while the controlling parameters for this system are Reynolds and Rayleigh number, the parameters of interest when considering swirling atmospheric flows are typically the Rossby number (relating inertial to Coriolis forces) and the Ekman number (relating viscous to Coriolis forces). The Ekman number is related to the reciprocal of the Reynolds number, but the Rossby number, which relates the angular velocity of the model polar vortex to the background rotation, is flow-dependent.

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Nomenclature

c_p	fluid specific heat capacity
E	Ekman number
f	Coriolis parameter
F_T	absolute base heat flux
g	gravitational acceleration
$\mathbf{g}$	gravity vector
$\hat{\mathbf{g}}$	unit vector in direction of gravity
h	boundary layer thickness scale
H	depth of fluid in tank
L	planar horizontal convection base length
N	Brunt–Väisälä frequency
Nu	Nusselt number
p	pressure
Pr	Prandtl number
Q	$2Re/Ra^{2/5}$
r	radial coordinate
R	tank radius
R_d	Rossby radius of deformation
Ra	Rayleigh number
Re	Reynolds number
t	time
T	temperature
T_w	wall temperature
$\mathbf{u}$	velocity vector
u_r	radial velocity
u_z	axial velocity
u_θ	azimuthal velocity
$u_{\theta,\mathrm{rel}}$	azimuthal velocity relative to tank
$V_{\max}$	maximum radial/horizontal boundary layer velocity
x	Cartesian horizontal coordinate
z	axial coordinate

Greek symbols

α	volumetric expansion coefficient
γ	power-law exponent
δT	radial temperature difference over base
ΔT	temperature scale (base flux)
δ_T	thermal boundary-layer thickness
δ_U	velocity boundary-layer thickness
κ_T	fluid thermal diffusivity
ν	fluid kinematic viscosity
θ	azimuthal coordinate
ω_θ	azimuthal component of vorticity
Ω	angular velocity of tank

For non-rotating horizontal convection, at high Rayleigh number, Rossby [12] demonstrated that the horizontal thermal layer has a thickness proportional to $Ra^{-1/5}$. However, in system undergoing strong rotation, the thinnest horizontal boundary layer is the Ekman layer. Therefore, the ratio of these layers will be important in describing the flow.

The effects of rotation on horizontal convection have been investigated by Stern [2], Hignett et al. [13], Barkan et al. [14] in various enclosure configurations. The dynamics of a horizontal convection in a rotating annuls was investigated experimentally by Hignett et al. [13]. The enclosure was rotated around its central axis in which a radial temperature gradient was maintained along the lower boundary in all direction from the axis. The dynamics of the flow was described in terms of a non-dimensional parameter Q, defined as the square of the ratio of the non-rotating thermal layer scale to the Ekman layer scale. Their experiments focused on the rotating regime with $Q \sim O(1)$. For a large Rayleigh number, six flow regimes were determined depending on the magnitude of parameter Q. They found that for small Q ratio, the flow is only weakly modified by rotation, and the scaling law for heat flux and thermal boundary layer is similar to non-rotating case (i.e. Rossby scaling for horizontal convection). Also, a critical value of $Q_c \approx 3.4$ were determined beyond which baroclinic instability and waves were predicted.

Park and Whitehead [1] conducted a set of laboratory experiments of rotating horizontal convection in a rectangular tank rotated around its axis to investigate moderate rotation rates as a model of oceanic meridional overturning circulation. They proposed a scaling law for the lateral heat flux and thermal boundary layer. When the typical values of the North Atlantic Ocean are introduced, their scaling law predicts heat flux comparable to that estimated by the North Atlantic when the vertical eddy diffusivity of heat is about 1 cm^2/s.

Barkan et al. [14] performed direct numerical simulations of rotating horizontal convection in a rectangular enclosure with an arbitrary axis of rotation for $Q \gg 1$. They extended the previous studies by exploring the rapidly rotating regime (i.e. $Q \gg 1$), which is more relevant to Earth's oceans. They also discussed the generation of baroclinic eddies in this flow, as it is thought to play an important role in the dynamics of oceanic overturning circulation. Analysis extending the models of [15,16] applied to the rapidly rotating case did an excellent job of predicting the inclination of stratification in the interior. Their results demonstrated that rapid rotation and baroclinic instability significantly modify the steady-state compared to non-rotating horizontal convection and therefore are essential components for the model of the overturning circulation and thermal structure of the ocean. Barkan et al. [14] briefly considered lateral variation of the Coriolis effect, varying the Coriolis frequency linearly from zero at one side wall. They demonstrated that for $Q = O(1)$, the same scaling behaviour was observed between constant-Q and laterally varying Q cases. Our system also provides a constant Coriolis forcing term throughout the enclosure, but the imposition of a radial horizontal thermal forcing distinguishes the present setup from previous models. Hence we use the term "radial horizontal convection" to describe the radial imposition of thermal horizontal convection forcing in all directions from an axis.

For horizontal convection in a rotating system, previous studies [17,2,1] proposed scalings law for the lateral heat flux and thermal boundary layer. These are linear scalings that disregard the effects of baroclinic eddies [14]. Robinson and Stommel [17], Park and Whitehead [1] used a scaling developed based on a geostrophic balance (rotation and pressure terms balanced) in the horizontal momentum equation and a balance between advection and vertical diffusion in the buoyancy equation [17,14], whereas Stern replaced the buoyancy-equation balance with a balance in the energy equation between buoyancy flux and molecular dissipation. This assumed that the flux was dominant throughout the thermal boundary layer while dissipation was confined to the Ekman layer.

The goal of the present study is to characterise the axisymmetric flow within a radial horizontal convection system subjected to rotation, and the associated heat transfer as a function of Reynolds and Rayleigh numbers.

The paper is organised as follows. The mathematical formulation and problem definition are given in Section 2, which also presents the governing equations and parameters. A scaling analysis yielding important relationships for the convective flow and heat transfer on the forcing boundary is presented in Section 3. The