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Analysis of thermal efficiency via analysis of heat flow and entropy generation during natural convection within porous trapezoidal cavities

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ABSTRACT

Thermal management via distributions of heatlines and entropy generation for natural convection within trapezoidal cavities in presence of hot left wall, cold right wall and adiabatic horizontal walls has been studied in this article. Heat flow visualization has been carried out via heatline concept. Galerkin finite element method has been used to analyze streamlines, isotherms, heatlines, entropy generation due to fluid friction and heat transfer over wide range of parameters ($10^{-5} \leqslant Da \leqslant 10^{-3}, 0.015 \leqslant Pr \leqslant 1000$ at $Ra = 10^6$). At low Darcy number ($Da = 10^{-5}$), conduction dominant heat transfer is found based on low magnitudes of streamlines and heatlines. Heatlines indicate that heat transfer occurs from hot left wall to cold right wall and thermal mixing is found inside the cavity. The thermal mixing is enhanced as Da increases from 10^{-5} to 10^{-3}. The thermal gradients are high near the lower portion of left wall and near upper portion of right wall for $Da \geqslant 10^{-4}$ irrespective of φ and Pr and thus, thermal boundary layer thickness is small along those zones. The maximum entropy generation due to fluid friction ($S_{\psi,max}$) occurs along the left wall for $\varphi = 30^\circ$ and 90° irrespective of Pr whereas that occurs along the right wall for $\varphi = 60^\circ$ at $Da = 10^{-3}$. The maximum entropy generation due to heat transfer ($S_{\theta,max}$) occurs at the left edge of bottom wall irrespective of Pr and Da for $\varphi = 30^\circ$ and 60° whereas that occurs at the left edge of bottom wall and right edge of top wall for $\varphi = 90^\circ$ with $Da = 10^{-5}$ and 10^{-4}. At $\varphi = 90^\circ$ with $Da = 10^{-3}$, $S_{\theta,max}$ occurs along both side walls for $Pr = 0.015$ whereas that occurs along left wall for $Pr = 1000$. It is found that total entropy generation is high for $Pr = 1000$ compared to that of $Pr = 0.015$ at higher Da. It is also found that the trapezoidal cavities with $\varphi = 60^\circ$ and 90° correspond to less entropy generation with significant heat transfer rates at $Da = 10^{-3}$ for $Pr = 0.015$ and $Pr = 1000$ and thus the trapezoidal cavities with $\varphi \geqslant 60^\circ$ may be the optimal design for thermal processing of $Pr = 0.015$ and $Pr = 1000$ fluids.

1. Introduction

The study of natural convection heat transfer has been considered as one of the most important research topics due to its wide range of applications in the engineering and physical problems, some of which include heat exchangers [1], food [2], solidification [3], ventilation [4] and electronic cooling [5] etc. Extensive investigations have been carried out on natural convection heat transfer by earlier researchers [6–11].

Several investigations on natural convection within porous enclosures have also been appeared in recent literature to display various applications [12–16]. Badruddin et al. [12] investigated the natural convection flows in a porous square annulus where the inner walls of the annulus are heated isothermally and the outer surfaces are exposed to cool temperature. Badruddin et al. [13] also studied the natural convection flows within square porous annulus where the inner walls of the annulus are maintained at cold and outer walls are exposed to hot temperature. Khandelwal et al. [14] studied natural convective flows within a rectangular enclosure where all the walls of the enclosure are adiabatic except the bottom wall, which is partially heated and cooled by sinusoidal temperature profile. Bagchi and Kulacki [15] studied natural convection heat transfer in fluid-superposed porous layers heated locally from below. Sankar et al. [16] analyzed the natural

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Nomenclature

Be	Bejan number
Da	modified Darcy number
g	acceleration due to gravity ($m\,s^{-2}$)
H	heatfunction
k	thermal conductivity, ($W\,m^{-1}\,K^{-1}$)
L	side of the trapezoidal cavity (m)
N	total number of nodes
$\overline{Nu}$	average Nusselt number
p	pressure, (Pa)
P	dimensionless pressure
Pr	Prandtl number
Ra	modified Rayleigh number
S	dimensionless entropy generation
$S_{\theta,total}$	dimensionless entropy generation due to heat transport
$S_{\psi,total}$	dimensionless entropy generation due to fluid friction
S_{total}	dimensionless total entropy generation
T	temperature of the fluid (K)
T_h	temperature of hot wall (K)
T_c	temperature of cold wall (K)
u	x component of velocity ($m\,s^{-1}$)
U	x component of dimensionless velocity
v	y component of velocity (ms^{-1})
V	y component of dimensionless velocity
X	dimensionless distance along x coordinate
x	distance along x coordinate (m)
Y	dimensionless distance along y coordinate
y	distance along y coordinate (m)

Greek symbols

α	thermal diffusivity ($m^2\,s^{-1}$)
β	volume expansion coefficient (K^{-1})
γ	penalty parameter
θ	dimensionless temperature
ν	kinematic viscosity ($m^2\,s^{-1}$)
ρ	density ($kg\,m^{-3}$)
φ	inclination angle with the positive direction of X axis
Φ	basis functions

convection flows in a vertical annulus filled with porous medium where the inner wall is subject to discrete heating, outer wall is maintained isothermally at a lower temperature, while the top and bottom walls and the unheated portions of the inner wall are kept adiabatic. Also, a number of numerical investigations based on natural convection heat transfer in porous square [17–19] and porous trapezoidal [20–22] enclosures are available in literature. However significant number of earlier works involve numerical results of convective heat transfer which were analyzed with the help of streamlines and isotherms. Although streamlines can adequately explain fluid flow, but isotherms are inadequate to study energy flow or the heat flux and its distributions. In the current work, analysis of heat flow in porous medium for differentially heated trapezoidal cavities using heatlines concept is presented. Kimura and Bejan [23] and Bejan [24] introduced heatline concept to visualize the heat flow in two dimensional convective heat transfer process.Thereafter, a number of work has been done to analyze the heat flow patterns for various situations [25–30].Current work also focuses on heat flow management and thermal efficiency based on second law of thermodynamics.

Although, the above investigations were carried out to understand the flow and isotherm patterns within enclosures, those studies were not adequate to explain the thermal efficiency of the system. In order to improve the system thermodynamically, Bejan [31–33] introduced a concept called as entropy generation minimization which is based on second law of thermodynamics. Entropy generation minimization results in minimum irreversibilities associated with the process and thus the overall efficiency of the system is increased. Therefore by analyzing the entropy generation due to heat transfer and fluid flow irreversibilities, the strategies to optimize the process may be achieved to increase the overall efficiency of the system. Several investigations have been carried out on the analysis of entropy generation minimization within square/rectangular cavities [34–37] and various complex geometries [38–42]. Soleimani et al. [34] used RBF-DQ method to analyze the entropy generation due to heat transfer within square cavity with two cases: In case 1, left wall is heated, right wall is maintained at constant cold temperature and horizontal walls are adiabatic, whereas case 2 corresponds to the partially heated bottom and left walls and partially cooled top and right walls. Delvar et al. [35] carried out the entropy generation analysis within a square cavity in which the left and top walls are isothermally cooled. The heater is located at the right wall of the cavity and remaining parts of right wall and bottom wall are adiabatic. Kaluri and Basak [36] analyzed the entropy generation during natural convection in porous square cavities that are heated differentially and discretely. Oliveski et al. [37] analyzed entropy generation during natural convection process within rectangular cavities where the left wall is hot and right wall is maintained at constant cold temperature while the horizontal walls are adiabatic. Heidary et al. [38] analyzed entropy generation for natural convection within an inclined porous square cavity whereas the left wall of the cavity is heated with a sinusoidal function and the right wall is cooled isothermally while horizontal walls of the cavity are adiabatic. Ziapour and Dehnavi [39] carried out the entropy generation analysis in Γ-shaped enclosure with circular corners. Kuo et al. [40] performed the entropy generation analysis for the free convection film condensation on an inclined porous elliptical tube. Zonouz and Salmanpour [41] used entropy generation minimization method to optimize the heat transfer rate from a curved vertical hot wall in a natural convection flow field. Varol et al. [42] studied entropy generation due to natural convection and flow in a triangular enclosure which is heated from below and symmetrically cooled from sloping walls. Basak et al. [43] analyzed the entropy generation during natural convection within fluid filled trapezoidal cavities for two cases where the bottom wall is uniformly heated in case 1, and that is nonuniformly heated in case 2. In both the cases side walls are cold and top wall is adiabatic. Basak et al. [44] also extended their study on the entropy generation for uniformly heated bottom wall and insulated top wall with linearly heated side walls (case 1) and linearly heated left wall with cold right wall (case 2). However, comprehensive analysis on the role of thermal mixing to entropy generation during natural convection within porous trapezoidal cavities in presence of hot and cold side walls with adiabatic horizontal walls is yet to appear in literature.

The objective of the present investigation is to analyze the heat flow visualization via heatline approach and entropy generation during natural convection within porous trapezoidal cavities whereas the left wall is hot and right wall is maintained at constant cold temperature while the top and bottom walls are adiabatic. Differentially heated cavity has received significant attention due