International Journal of Heat and Mass Transfer 77 (2014) 140–146

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Double tree structure in a conducting body

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ARTICLE INFO

Article history:
Received 7 April 2014
Received in revised form 28 April 2014
Accepted 30 April 2014
Available online 5 June 2014

Keywords:
Constructal
Tree structure
Vascular design
Ground coupled
Heat pump

ABSTRACT

In this paper we consider the fundamental problem of how to design the spacing between two plane and parallel tree structures buried in and transferring heat to a conducting medium. The Y and T tree-shaped structures are configured as two palms facing each other. The search for the optimal spacing is pursued in solid domains with three sizes. First, we studied a cube where the tree palms structure grow to the second level bifurcation. Second and third, we considered a small cube and a large cube where the tree structure consists of just the first level bifurcation. The results show that the best spacing between palms is half of the side of the cube, in order to avoid the interference between the two tree and adiabatic boundaries of the cube in which the trees structures are embedded. The spacing is possible to optimize, without being affected by the shape of the structure, the bifurcation level of the structure and the size of cube. The decision of optimal spacing is not affected by changes in the volume fraction occupied by tree structure in the conducting body.

1. Introduction

The generation of flow configuration in nature and engineering is governed by the constructal law [1], which accounts for the universal tendency of freely morphing flow systems to generate configurations that evolve toward greater access for their currents. Reviews of the progress made with the constructal law of design in nature and engineering are provided in Refs. [2,3]. Tree-shaped designs emerge naturally when the tendency of the flow is to flow more easily between areas (or volumes) and discrete points.

One of the current applications of constructal design is to improve the performance of the ground coupled heat pump systems. Improvements have been documented with classical single-pipe designs shaped as hairpins and serpentines [4–14], and more recently with tree-shaped designs [15–17].

In this paper we propose a double tree design, Fig. 1. The trees face each other like two "palms" connected at the finger tips. We consider the effect of the geometric aspect of the structure on the overall thermal contact between structure and ground. Special attention is paid to the spacing between the two palms.

2. Numerical model and method

The links of the tree structure are modeled as isothermal cylinders (pipes) buried in a solid at a different temperature. The spacing between pipes in each design is S, Fig. 3. The solid medium is shaped as a cube with the volume $V = 4L_1 \times 4L_1 \times 4L_1$, where L_1 is the length of the trunk in the T- and Y-shaped tree configurations. The boundary of this volume is modeled as adiabatic. The cube is initially at a temperature (T_1) that is higher than the temperature of the cylinders (T_0). In time, a cooled zone grows around the cylinders. The following analysis applies equally to the reverse situation where the buried structure is warmer than the solid.

As an approximation we assume that the pipe flow is sufficiently intense so that the pipes can be modeled as isothermal. This is a good model because the seasonal time of heat pump operation is much longer than the time of the fluid traveling from inlet to outlet in the duct.

The tree is an assembly of Y-shaped and T-shaped bifurcations. The lengths of the branches are sized in the sequence

$$L_1, \quad L_2 = \frac{L_1}{2}, \quad L_3 = L_2, \quad L_4 = \frac{L_3}{2}, \quad L_5 = L_4 \tag{1}$$

All the channels cross sections are round. The minimization of fluid resistance calls for a particular sequence of diameter sizes, depending on whether the flow regime is laminar or turbulent [3]. For the

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.04.072
0017-9310/

Nomenclature

D	channel diameter, m
L	channel length, m
N	number of bifurcation levels
S	spacing between pipe centers, m
T	temperature, K
t	time, s
U	mean velocity, m s^{-1}
V	volume, m^3
x, y, z	coordinate, m
Greek Symbols	
α	thermal diffusivity, m^2 s^{-1}
β	bifurcation angle
θ	dimensionless temperature
φ	composite volume fraction (porosity)
Subscripts and superscripts	
avg	average
$*$	dimensionless

laminar regime the diameters of the channels are sized relative to one another in accordance with the Hess-Murray rule [3],

$$\frac{D_i}{D_{i+1}} = 2^{1/3} \quad (i = 1, 2, \ldots) \tag{2}$$

The trunk has the diameter D_1. Unlike the T-shaped design, the angles between branches are allowed to vary, and this gives the design freedom to morph. The first bifurcation angle between the trunk and the first branch is β_1. The second, third and fourth bifurcation angles are β_2, β_3, etc. This morphing architecture was optimized numerically, and its geometric features are summarized in Fig. 2. The temperature field was simulated as time-dependent heat conduction by using a computational package [18]. We tested the independence of the results with respect to mesh size. The changes in the numerical results were less than 1 percent when the number of mesh elements was increased in steps of 50 percent. The conservation of energy in the solid that surrounds tree is governed by

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \tag{3}$$

where α is thermal diffusivity of the solid, and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, where x, y, and z are defined in Fig. 3. For greater generality, we determined the temperature field in terms of the dimensionless variables

$$(x_*, y_*, z_*) = (x, y, z)/V^{\frac{1}{3}} \tag{4}$$

$$t_* = \alpha t / V^{\frac{2}{3}} \tag{5}$$

$$\theta = \frac{T_0 - T}{T_0 - T_1} \tag{6}$$

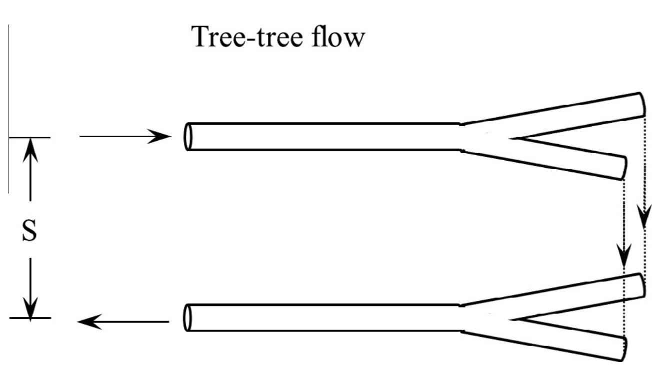

Fig. 1. Tree-tree flow design.

where T_0 is the lowest temperature (the tree surface). The length scale of the configuration is $V^{1/3}$. Written in terms of dimensionless variables, Eq. (3) becomes

$$\frac{\partial \theta}{\partial t_*} = \frac{\partial^2 \theta}{\partial x_*^2} + \frac{\partial^2 \theta}{\partial y_*^2} + \frac{\partial^2 \theta}{\partial z_*^2} \tag{7}$$

The initial and boundary conditions are

$$\theta = 0 \text{ at } t_* = 0 \tag{8}$$

$$\frac{\partial \theta}{\partial x_*} = 0 \text{ at } x_* = 0, 4 \quad \frac{\partial \theta}{\partial y_*} = 0 \text{ at } y_* = -2, 2 \quad \frac{\partial \theta}{\partial z_*} = 0 \text{ at } z_* = 0, 4 \tag{9}$$

3. Spacing of tree-tree structure in a cube

Beginning with $t_* = 0$, heat flows from the solid cube into the tree-shaped sink. In time, the volume averaged temperature of the cube decreases and approaches T_0. We are interested in the effect of the tree structure spacing on the approach to thermal equilibrium. We seek the geometric configuration that facilitates the approach to thermal equilibrium. For this purpose, we considered the duration of the heat transfer process fixed at $t_* = 1$, and investigated the effect of the tree structure spacing on the average temperature of the cube at that time.

$$\theta_{avg} = \frac{T_\infty - T_{avg}}{T_\infty - T_{in}} \tag{10}$$

Consider first the heat transfer performance of the Y and T shaped design when the tree evolution occurs in the presence of first bifurcation level ($N = 1$). Fig. 4a and b show the effect of the tree-tree spacing on the dimensionless average temperature θ_{avg}, Eq. (10). The curve has a clear maximum at $S = 2L_1$. This conclusion is due to the interference between the two trees and the adiabatic boundaries of the cube in which the tree structures are embedded.

Fig. 5 shows the comparison of the pipes shape (Y, T, and U) on the thermal performance, for the same porosity and surface area. In Fig. 5, the spacing between the two trees is set at $S = 2L_1$. In an earlier study [14] we found that the spacing for the U-shaped design should not be smaller than $S/D = 5$, therefore in Fig. 5 we set $S/D = 5$. The steeper curve represents the design with better thermal contact between architecture and conducting medium, and the better design is the Y-shape structure. Next, we varied the spacing between pipes when a tree evolution of Y and T is in second bifurcation level ($N = 2$). Fig. 6a and b also show that the optimal spacing is $S = 2L_1$. The optimal spacing is equal to half of a side length of the cube that tree structure is embedded.