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The effect of unsteadiness on mixed convection boundary-layer stagnation-point flow over a vertical flat surface embedded in a porous medium

Haliza Rosali [a], Anuar Ishak [b], Roslinda Nazar [b], J.H. Merkin [c,*], I. Pop [d]

[a] Department of Mathematics, Faculty of Science, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia
[b] Centre for Modelling and Data Analysis, School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia
[c] Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, UK
[d] Department of Applied Mathematics, Babeş-Bolyai University, R-3400 Cluj CP 253, Romania

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ABSTRACT

The unsteady mixed convection boundary-layer flow towards a stagnation point on a heated vertical surface embedded in a porous medium is considered. The governing partial differential equations are reduced to a system of nonlinear ordinary differential equations using a similarity transformation. These are then solved numerically using the Keller-box method. These numerical results are complemented by asymptotic solutions for large values of A. The influence of dimensionless parameters λ (mixed convection), A (time variation), n (wall temperature variation) on the flow field and heat transfer is analyzed and discussed, dual solutions being found for both assisting and opposing buoyant flows.

1. Introduction

The study of the convective flow and heat transfer in a layer of saturated porous material has many practical applications in engineering such as building thermal insulation, geothermal systems, food processing and grain storage, solar power collectors, contaminant transport in groundwater, casting in manufacturing processes, drying processes, nuclear waste, etc. Representative studies on this subject can be found in the recent monographs by Ingham and Pop [1], Pop and Ingham [2], Vafai [3,4], Vadasz [5] and Nield and Bejan [6].

The problem of mixed convection boundary-layer flow in fluid-saturated porous media has attracted many investigations since the results of these studies have a great technical importance. Cheng [7] seems to be the first to consider the problem of steady mixed convection in porous media along an inclined surface. Merkin [8] later considered the mixed convection boundary layer flow about a vertical flat surface embedded in a saturated porous medium. More recently Ishak et al. [9] treated the problem of mixed convection boundary-layer flow through a stable stratified porous medium bounded by a vertical surface. The effects of thermal stratification on the surface shear stress and the surface heat transfer were discussed. Mixed convection boundary-layer flows near the lower stagnation point on a vertical flat plate embedded in a fluid-saturated porous medium characterized by an anisotropic permeability has been discussed by Bachok et al. [10]. Chandrasekhara [11] investigated the problem of mixed convection about a horizontal surface embedded in a porous medium where the buoyancy force acts perpendicular to the surface.

Many authors have considered various aspects related to this basic problem but most of these studies deal with steady state flow. However, the flow may be unsteady due to sudden changes in the external stream or in the surface temperature. An analysis of the unsteady mixed convection boundary-layer flow on a vertical flat surface embedded in a porous medium has been considered by Harris et al. [12]. Nazar et al. [13] have studied the problem of unsteady mixed convection boundary layer flow near the region of a stagnation point on a heated vertical surface embedded in a Darcian fluid-saturated porous medium. Also, Merrill et al. [14] studied the final steady flow near a stagnation point on a vertical surface in a porous medium. The problem of unsteady planar and axisymmetric stagnation-point flow of an incompressible viscous fluid over a surface moving along the direction of flow impingement has been investigated by Zhong and Fang [15]. However, Yang [16] was the first to consider the two-dimensional unsteady

* Corresponding author. Tel.: +44 1937584752.
E-mail address: amtjhm@maths.leeds.ac.uk (J.H. Merkin).

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Nomenclature

a	parameter associated with free stream velocity and wall velocity
A	time variation parameter
C_f	skin friction coefficient
f	dimensionless stream function
$h(t)$	position of the surface
m	parameter associated with planar ($m=0$) or axisymmetric flow ($m=1$)
u, v	velocity components in the x and y directions, respectively
p	pressure of the fluid flow
t	time
n	the temperature power index
K	permeability of the porous medium
k	thermal conductivity
Nu_x	local Nusselt number
q_w	surface heat flux
T	fluid temperature
T_w	surface temperature
T_∞	ambient temperature
T_0	temperature scale, measure of the difference $T_w - T_\infty$
u_e	velocity of the external flow
v_w	mass transfer velocity
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek letters

α_m	effective thermal diffusivity
β	thermal expansion coefficient
γ	parameter associated with free stream velocity and plate velocity
η	similarity variable
θ	dimensionless temperature
λ	mixed convection parameter
ν	kinematic viscosity
μ	dynamic viscosity
σ	the heat capacity ratio of the fluid-filled porous medium
ψ	stream function
τ_w	surface shear stress

Subscripts

w	condition at the surface
∞	ambient conditions

stagnation-point flow towards a stationary surface. Many studies have been conducted on the unsteady flow as discussed in [17–19]. Motivated by the above investigations, the present paper considers the unsteady mixed convection boundary-layer flow near the lower stagnation point on a heated vertical surface embedded in a porous medium. A similarity transformation is applied to reduce the governing partial differential equations to a system of ordinary differential equations. The characteristics of the flow and heat transfer are reported for both assisting and opposing flows.

One feature to note about our model is that the transpiration velocity v_w reflects the form of the outer flow and thus reflects the unsteadiness in both the outer flow and the wall temperature. This is different to a related problem that has been treated by [25], where a more general form for the transpiration velocity, independent of the unsteadiness, is assumed. In fact this direct connection of the transpiration velocity with the outer flow is a new feature and, as far as we are aware, not been considered previously.

2. Problem formulation

We consider the unsteady mixed convection boundary-layer flow near the lower stagnation point on a heated vertical surface embedded in a porous medium. It is assumed that the stagnation point is at $(0, h(\bar{t}))$ as shown in Fig. 1, where $\bar{t}$ is time, $h(\bar{t})$ is the position of the surface, x and y are Cartesian coordinates measured respectively along the surface and normal to it. We assume that the plate coincides with the surface $y=0$ and the flow takes place in the region $y \geqslant 0$. The mass transfer velocity through the surface is $v_w(\bar{t}, x)$ and the flow far from the surface is $u_e(\bar{t}, x)$. We also assume that the temperature of the surface is $T_w(\bar{t}, x)$ and that of the ambient fluid is constant at T_∞. The forms of $u_e(\bar{t}, x), T_w(\bar{t}, x)$ and $v_w(\bar{t}, x)$ will be specified below. Under the assumptions of Darcy's law and the Boussinesq approximation, the basic unsteady boundary-layer equations for the problem under consideration are, see Nield and Bejan [6] for example,

$$\frac{1}{x^m}\frac{\partial(x^m u)}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u = u_e + \frac{gK\beta}{\nu}(T - T_\infty) \tag{2}$$

$$\frac{1}{\sigma}\frac{\partial T}{\partial \bar{t}} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and v are the velocity components along the x and y axes, T is the fluid temperature, K is the permeability of the porous medium, g is the acceleration due to gravity, ν is the kinematic viscosity, α_m is the effective thermal diffusivity and σ is the heat capacity ratio of the fluid-filled porous medium to that of the fluid. In Eq. (1), $m=0$ is for planar flow and $m=1$ is for axisymmetric flow. We assume that Eqs. (1)–(3) are subject to the initial and boundary conditions

$$u = v = 0, \quad T = T_\infty, \quad \text{at} \quad t = 0 \quad (x, y \geqslant 0) \tag{4}$$

$$v = v_w, \quad T = T_w \quad \text{on} \quad y = 0, \quad u \to u_e, \quad T \to T_\infty \quad \text{as} \quad y \to \infty \quad (t > 0) \tag{5}$$

To eliminate σ from Eq. (3), we put $t = \bar{t}/\sigma$ and we assume that $u_e(t, x)$ and $T_w(t, x)$ take the form

$$u_e(t, x) = \frac{ax}{1+\gamma t}, \quad T_w(t, x) = T_\infty + T_0\frac{x^n}{1+\gamma t} \tag{6}$$

where T_0 is a characteristic temperature scale with $T_0 > 0$ for a heated plate (assisting flow) and $T_0 < 0$ for a cooled surface (opposing flow). Also, $a > 0, n$ and $\gamma > 0$ are constants. The form of time dependence used in (6) was originally suggested by Yang [16] and Birkhoff [24] and is usually referred to as 'hyperbolic time variation'.

Following Zhong and Fang [15], we look for a similarity solution of Eqs. (1)–(3) by introducing the transformation

$$\psi = \sqrt{\frac{a\alpha_m}{1+\gamma t}}x^{1+m}f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = y\sqrt{\frac{a}{\alpha_m(1+\gamma t)}} \tag{7}$$

where ψ is the stream function, defined in the usual way as $u = \frac{1}{x^m}\frac{\partial\psi}{\partial y}, v = -\frac{1}{x^m}\frac{\partial\psi}{\partial x}$. The continuity Eq. (1) is automatically satisfied, and we then have