International Journal of Heat and Mass Transfer 77 (2014) 157–172

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Scale-up on mixing in rotating microchannel under subcritical and supercritical operating modes

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ARTICLE INFO

Article history:
Received 7 February 2014
Received in revised form 7 May 2014
Accepted 12 May 2014
Available online 5 June 2014

Keywords:
Centrifugal microfluidics
Scale-up
Mixing
Crossflow
Supercritical and subcritical modes
Coriolis

ABSTRACT

The factors affecting flow and mixing of two different fluids in a rotating radial microchannel are investigated using numerical simulation, experiments, analytical approach and dimensional analysis. As has been verified by both numerical simulation and experiments, depending on the channel width-to-height aspect ratio there are two distinctly different modes of operation, subcritical and supercritical modes, that yield identical mixing quality with the supercritical mode providing higher volumetric throughput. Four dimensionless groups (namely, rotational Reynolds number, channel length-to-height, channel width-to-height, and channel initial radial-location-to-height) are found to be important for correlating the quality of mixing of fluids in the rotating microchannel operating in either mode in form of scale-up laws. The latter, which fill the badly needed missing knowledge gap, are useful for design, operation, and optimization of rotating microchannels for fluid mixing.

1. Introduction

Fast mixing is of primary importance in a variety of microfluidics-based applications. For fast chemical reaction and analysis, this requires the test sample and reactants, which initially stored separately in their individual reservoirs, to be fully uniformly mixed during the course of flowing through a microchannel to the reaction or analysis chamber. It is challenging to achieve fast mixing as the flow in a microchannel is dominated by viscous resistance force at small Reynolds number (*Re*) and the flow duration in the channel is relatively short [1]. Due to the absence of chaotic motion, mixing of different chemical species/reactants relies primarily on the slow molecular diffusion. Besides diffusion can be somewhat enhanced by increasing the process temperature, diffusive mixing can be optimized by increasing the concentration gradient and the interfacial/contact surface area [2]. The latter requires breaking up the inhomogeneous fluids into smaller zones with increasing interfacial area between them facilitating diffusion to complete the mixing process.

Numerous active and passive micromixers have been developed to achieve this goal. The passive micromixers have attracted increasing interest as they do not require external actuation power or moving parts, and some of their exemplary principles and devices are summarized in Table 1. Among the passive micromixers, centrifugal microfluidics [21,23–28] offer several advantages: (a) it is capable of controlling and releasing fluids by centrifugal pumping and capillary valving without additional pumps and valves; (b) the device can be easily fabricated and readily incorporated into a lab-on-chip system without the need of complicated three-dimensional structure; (c) the disk can be fabricated economically for disposal applications by mass-production from inexpensive materials, such as polycarbonate. The results of the present study are extendable to various other applications. As an example, rotating channels and passages have been widely employed for internal convective cooling of turbine blades [29–31].

Rotating channels and passages can also be used to improve mixing. For the rotating unobstructed radial channel, Chakraborty et al. [17] discussed the three regimes of mixing in rotating microchannel, namely, a diffusion-based mixing regime at low rotation speed, a Coriolis-based mixing regime at intermediate rotation speed, and an instability-based mixing at high angular speed. They have correlated the mixing quality and the channel length, yet their study was limited to a fixed microchannel width and height. Haeberle et al. [18] investigated the Coriolis-induced crossflow in reshaping and increasing the interface separating the two different fluids to achieve mixing. They also showed that improved mixing quality can be reached at high flow rate, or more accurately higher rotation speed. Apart from the flow rate or rotation speed, the effect of channel geometry, i.e. width, height and length, on mixing

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.05.011
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Nomenclature

A	cross section area (m^2)
a	constant (–)
b	constant (–)
b'	constant (–)
C	concentration (mol/m^3)
D	diffusion coefficient (m^2/s)
H	height of channel (m)
I	intensity (–)
L	length of channel, or length along channel (m)
l	diffusive mixing length (m)
L_0	distance between the inlet of the channel and the axis of rotation (m)
Pe	Péclet number (–)
P	pressure (Pa)
Q	flow rate (m^3)
Re	rotational Reynolds number (–)
t	time variable (s)
U	averaged throughflow velocity (m/s)
u	throughflow velocity along radial direction (m/s)
V	reservoir volume (m^3)
v	crossflow velocity (m/s)
W	width of channel (m)
x	radial coordinate (m)
y	transversal coordinate (m)
z	axial coordinate (m)

Greek letters

α	mixing quality (–)
μ	dynamic viscosity (kg/m s)
υ	kinematic viscosity (m^2/s)
ζ	radial coordinate along the channel (m)
ρ	density (kg/m^3)
σ	standard deviation (–)
ϕ	dimensionless parameter (–)
Ω	rotation speed (rad/s)
ψ	dimensionless parameter (-)

Subscripts

av	average
cor	Coriolis
crit	critical
i	grid or pixel location
max	maximum value
min	minimum value
m	mixing
N	normalized value

Table 1
Summary of representative investigations on passive type micromixers.

Categories			Mixing principles	Investigators
T- and *Y*-flow configurations			Bi-lamination	Bothe et al. [3]; Engler et al. [4]; Wong et al. [5]
Alternate-feed flow configurations (i.e. bifurcation)			Multi-lamination	Bessoth et al. [6]
Hydrodynamic focusing			Parallel lamination by sheath flow to reduce diffusion length	Nguyen et al. [7]; Park et al. [8]
Serpentine/zigzag/spiral channel			Chaotic advection	Liu et al. [9]; Liu et al. [10]; Yang et al. [11]
Embedded barriers			Split and recombination	Lee et al. [12]; Ansari et al. [13]
			Chaotic advection	Stroock et al. [14]; Bhagat et al. [15]; Park et al. [16]
Centrifugal microfluidics	Rotating channel (continuous mode)	Unobstructed radial channel	Coriolis effect	Chakraborty et al. [17]; Haeberle et al. [18]; Ducrée et al. [19]
		Obstructed radial channel	Coriolis effect and multi-lamination	Ducrée et al. [20]
		Zigzag channel	Coriolis, and centrifugal effect	Ren and Leung [21]
	Rotating chamber (batch mode)		Inertial and Coriolis effect	Grumann et al. [22], Ren and Leung [23,24]

quality has however not been investigated. Ducrée et al. [19] investigated in a limited scope the dependence of mixing on the channel length as well as the effect on mixing for channels with different widths, 0.1, 0.25 and 0.5 mm, respectively, for a constant channel height of 0.1 mm. Their results demonstrated that mixing can be improved as the channel length increases. However, there is no quantitative correlation between mixing quality and the channel length. Further, they have only qualitatively demonstrated that mixing can be improved as the channel width increases. They argued that this is due to a reduction in "characteristic diffusion length", which is controversial for reasons stated below.

In fact, there are two different, conflicting scenarios regarding channel geometry effect to mixing, and these have never been adequately addressed by the aforementioned research. In the rotating channel, mixing is dependent on Coriolis effect at intermediate rotation speed [17]. The fluid is driven by the Coriolis force in form of crossflow from the leading to the trailing wall, and subsequently returning back to the trailing wall. It takes longer time to complete the circulation when the channel width is increased. This is in contrast to the findings of Ducrée et al. [19] that a wider channel is beneficial. On the other hand, when the channel width becomes too small, the flow turn-around effect (flow slows down as it approaches a wall and reaccelerates after it changes direction moving away from the wall) becomes more pronounced, which slows down crossflow resulting in poor mixing. This implies that wider channel is better for mixing. This second scenario is in agreement with Ducrée et al.'s observation [19] yet contradicts the first scenario. How can both opposing scenarios exist at the same time? This paradox has also been observed by Roy et al. who investigated the effect of channel width-to-height aspect ratio and rotation on the secondary flow numerically and reported a width-to-height aspect ratio dependent critical rotational Reynolds number to demarcate the dominance of rotational effect [32]. They found that the viscous diffusion is sufficiently strong to suppress the secondary flow in microchannels with very high, or very low, width-to-height aspect ratio. Consequently, they infer that best