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Analysis of sub- and supercritical Hopf bifurcation with a reduced order model in natural circulation loop

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ABSTRACT

The prime objective of the work is to carry out bifurcation analysis of a single channel natural circulation loop having high system pressure. The objective is achieved in two steps. First step is to perform linear stability analysis of the system with the state space approach. The analysis is used to obtain stability maps in N_{pch} and N_{sub} plane. In the second step, limited time series analysis is carried out numerically in the vicinity of the obtained stability boundaries. This analysis leads to delineation of bifurcation characteristics of the system namely subcritical and supercritical bifurcations. The existence of limit cycles and their behavior are also presented. The effects of different design parameters on the aforementioned bifurcation phenomena are shown. It is observed that, these bifurcation phenomena are quite sensitive to the design parameters. The reduced order model used in the present work has been developed by keeping a new insight to include the density variation in single phase and the drift flux correlations in the two-phase.

1. Introduction

Due to simplicity in design and passive mode of heat transfer, two-phase natural circulation loops are popular for several applications in nuclear and other process industries. Moreover, it helps to reduce the cost of the system and has lesser maintenance problems compared to forced circulation systems. However, there is possibility of occurrence of various kinds of instabilities which may result in serious problems in its operation. In a broad sense these are known as thermal hydraulic instabilities, which include periodic oscillations of flow rate, temperature and density of fluid as well as pressure drop. The instabilities are undesirable in aforementioned systems because continual flow oscillations can induce mechanical vibration to the constituent components thereby significantly affecting the performance of the system.

Unlike the forced convection system where the flow rate is controlled by applied external pressure drop, in a natural circulation system the flow rate is determined by the void generated due to heating and pressure loss during the flow. It is evident from past studies that a natural circulation system exhibit two-types of instabilities, classified as Type-I and Type-II [1–12]. The Type-I instability arises at low power conditions where the steam quality is low. During a low steam quality situation, a slight change in the quality due to any disturbance causes a large variation in void fraction and consequently the driving head changes. As a result the flow fluctuations occur. But at high power conditions, when the void fraction and steam quality is large, the frictional pressure losses increases. Moreover, a large void fraction generates void propagation time delay in the two-phase region of the system. Under these situations, any disturbances in the flow can cause large fluctuations in the fluid density as well as frictional pressure drops. These fluctuations eventually induce the coolant flow rate to oscillate, which is classified as Type-II (Density wave oscillation) instability. The existence of excursive and density wave instability zone in such boiling natural circulation loops have been demonstrated by Lee and Lee [1]. Kyung and Lee [3] using their analysis showed that, at higher subcooling, instead of continuous circulation, there exist three different modes of circulation, which they named as periodic circulation(A), periodic circulation(B) and multimode circulation. The Periodic circulation(A) appears at low heat flux conditions. This periodic oscillation is characterized by existence of incubation (no boiling) periods which is considered to be the geysering instability. The periodic circulation(B) appears at higher heat fluxes and is characterized by flow oscillations with continuous boiling inside the flow channel (no incubation period). During this oscillation, the void fraction fluctuates in the higher ranges, which is considered to be density wave oscillation. The final mode is multimode circulation. In this mode of oscillation, both the flow rate and void fraction oscillate with more than one period of oscillation.

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Nomenclature

A^*	cross section area (m^2)
B_t^*	loop width (horizontal section, m)
C_0	void distribution parameter
D_h^*	hydraulic diameter of flow channel (m)
D_{doc}^*	diameter of downcomer (m)
D_r^*	diameter of riser (m)
f	friction factor $\left(\frac{0.184}{R_e^{0.2}}\right)$
Fr	Froude number $\left(\frac{v_0^{*2}}{g^* L_{ch}^*}\right)$
g^*	acceleration due to gravity (m/s^2)
h^*	enthalpy (kJ/kg)
H_t^*	downcomer level (m)
j^*	volumetric flux (m)
J	Jacobian matrix
K_{inlet}	inlet pressure loss coefficient
K_{exit}	exit pressure loss coefficient
L_{ch}^*	total length of flow channel (m)
L_r^*	length of riser (m)
N_f	friction number $\left(\frac{fL_{ch}^*}{2D_h^*}\right)$
N_{pch}	phase change number $\left(\frac{q''^* \Delta\rho^* \xi_h^* L_{ch}^*}{A^* \Delta h_{fg}^* v_0^* \rho_g^* \rho_f^*}\right)$
N_{sub}	subcooling number $\left(\frac{(h_{sat}^* - h_{inlet}^*)\Delta\rho^*}{\Delta h_{fg}^* \rho_g^*}\right)$
N_{feed}	feed number $\left(\frac{(h_{sat}^* - h_{feed}^*)\Delta\rho^*}{\Delta h_{fg}^* \rho_g^*}\right)$
q''^*	wall heat flux (W/m^2)
t^*	time (s)
v_0^*	Reference velocity (m/s)
v_{inlet}^*	inlet velocity of coolant (m/s)
V_{gj}^*	average drift velocity (m/s)
x	steam quality
z^*	distance along the axis of flow channel (m)
ξ_h^*	heated perimeter (m)
α	void fraction
μ^*	boiling boundary (m)
ρ^*	density (kg/m^3)

Subscripts

1Φ	single phase
2Φ	double phase
doc	downcomer
ex	exit
f	liquid
g	vapor
in	inlet
m	mixture
r	riser

Superscripts

*	dimensional quantity
~	steady state value

Abbreviations

AHWR	advanced heavy water reactor
BWR	boiling water reactor
DFM	drift flux model
DWO	density wave oscillation
HEM	homogeneous equilibrium model
HPNCL	high pressure natural circulation loop
NCBWR	natural circulation boiling water reactor
SB	stability boundary

Lots of researches have been carried out to understand the aforementioned instabilities occurring in two-phase flow systems. Existing Reviews [13–17] reveal that density-wave instability is the most common type of instability occurring in boiling two-phase systems. To meet the requirement of analysis of density-wave instability, several models have been prescribed. Using homogeneous equilibrium model, linear analysis have been carried out by Lee and Lee [1], Furutera [18], Delmastro et al. [19], Wang et al. [20] and Nayak et al. [21]. Some investigations have also done for nonlinear analysis [7,22,23] in such systems. However, these studies suggest that though the application of homogeneous assumption makes the system equations relatively simpler, but the accuracy to determine stability boundaries have been compromised. Followed by this, linear stability analysis using a four-equation drift flux model has been carried out by Ishii and Zuber [24], Saha and Zuber [25], Park et al. [26], Rizwan-uddin and Dorning [27] and Van Bragt et al. [28].

As mentioned above, using a Laplace transformation approach, the linear analysis of flow instabilities in an open two phase natural circulation loop is presented by Lee and Lee [1]. The occurrence of excursive instability [3] was also investigated in the study. However, these studies were confined to analyze the linear stability behavior of the system, which are predominantly known as local stability characteristics, and did not depict the nonlinear behavior. It should be noted that using a Laplace transformation approach, though a linear analysis can be performed, but it is almost impossible to analyze the nonlinear behavior that defines the global stability. Later using a reduced order model [29], Lee and Pan [7,23] extended the study for a single channel and double channel loop respectively. In these studies, using a linear enthalpy profile, the conservation equations (PDEs) are integrated over the concerned regime, which results into a set of nonlinear ODEs and thus so called reduced order model. However, these nonlinear ODEs are solved simultaneously with an applied perturbation to obtain the stability characteristics of the system. Using a similar kind of Laplace transform approach, the linear stability analysis for a primary heat transport system used in Advanced Heavy Water Reactor (AHWR) has also been performed [2,5,6,8,21,30]. Followed by this, nonlinear numerical models [7,22,23] had been presented where researchers showed the different types of oscillation behavior in Type-I and Type-II region of stability map. Using a lumped parameter model, Durga Prasad and Pandey (2008) [10] and Durga Prasad et al. [9] also extended the study for a single channel and two-channel loop respectively. Goudarzi and Talebi [12] repeated the analysis using the same methodology as used by Nayak et al. [5]. All these aforementioned studies include the sensitivity analysis where the effects of various design parameters on flow stability have been demonstrated. It is noted that there are mainly two approaches, which have been extensively used earlier. First, Laplace transform [2,5,6,8,21,30], which is although quite useful, but is limited to linear analysis only. The second approach known as reduced order model [4,7,23], can be used to obtain nonlinear characteristics of the system. However, this approach was found to solve ODEs numerically over a wide range of parameter and hence is computationally intensive. Therefore most of the times, only a discontinuous stability map is obtained.

From the aforementioned studies it can be anticipated that although lot of researches have been carried out to know the stability behavior of a two-phase natural circulation loop, but only a few [7,9,10,23] are available that can delineate the nonlinear