International Journal of Heat and Mass Transfer 77 (2014) 369–376

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Analytical and numerical approach in the simple modelling of thermoacoustic engines

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ARTICLE INFO

Article history:
Received 7 March 2014
Received in revised form 28 April 2014
Accepted 9 May 2014
Available online 11 June 2014

Keywords:
Thermoacoustics
CFD analysis
Analytical solution

ABSTRACT

Thermoacoustic phenomena can be modelled by means of analytical closed-forms and numerical models. This paper includes results related to the modelling of the thermoacoustic engine operation. A simplified analytical model of the engine is presented and the obtained results are compared to the calculations performed for the numerical model using the Ansys CFX software package. A number of assumptions and simplifications adopted in both models are discussed in detail. The object of the comparisons is to determine the reliability of modelling of the thermoacoustic engine operation and to evaluate the suitability of the models for solving inverse problems or problems related to optimization.

1. Introduction

Thermoacoustic phenomena can be modelled by means of analytical and numerical models. Numerical calculations usually require great computational costs resulting from the need to perform non-stationary calculations. This also concerns modelling the operation of machines with a relatively unsophisticated geometry. As a result, if parameters need to be evaluated repeatedly, for example during an optimization process or feasibility study of the design process, it can be very difficult, if not impossible, to use this approach. Then the use of an analytical model is justified because in it the solution can be obtained in a significantly shorter time. Obviously, analytical models are usually based on considerable simplifications, which raises doubts as to their suitability for solving complex physical processes.

The number of works describing thermoacoustic phenomena from the analytical perspective only is relatively small. Detailed theoretical models can be found in already classic works by Swift [12,13] and Rott [10], as well as in [8]. In all these works equations are derived which relate acoustic and thermodynamic phenomena to each other. The theoretical model obtained in this way contributed to a better understanding of thermoacoustic phenomena and a deeper interest in such processes.

The technological potential and simplicity of thermoacoustic devices that the theoretical results point to ([5,6,12,20]) generate a growing interest in their application in practice. In most works that have been published in the past, closed-forms are used as auxiliary models to optimize or evaluate the results obtained by means of other methods. For example, in [1,14] analytical solutions are used to design and optimize parameters of the engine itself, whereas in [5] the analysis concerns the impact of the device structure and parameter selection on the processes occurring in it. Some researchers, even though they are not trying to find a solution using the analytical approach, compare the results they obtain (from the numerical model or from experimental testing) with theoretical results (from the computer code DeltaE for example) ([1,4]).

Generally the analytical solution to differential equations constituting the theoretical model presents numerous difficulties. Making appropriate assumptions, the analytical approach may result in finding the curves of thermoacoustic parameters (such as pressure and velocity) as a variable geometrical function related to the device length. In this case, it is necessary to determine the boundary conditions needed to solve the problem unequivocally [5,8,10,13].

It is quite obvious that the greatest handicap of theoretical models is their linearity. Due to that, it is impossible to capture non-linear effects such as turbulence generated in the stack area, which causes a non-linear distribution of temperature and has an impact on heat exchange conditions [2].

Apart from analytical models, numerical models based on in-house or commercial CFD codes are available, too. The problem of the Rijke tube operation [3] is a kind of this application based on

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.05.018
0017-9310/

Nomenclature

c	speed of sound, m/s
c_p, c_v	specific heat, J/(kg $*$ K)
i	imaginary unit
k	thermal conductivity, W/(mK)
p	pressure, Pa
Pr	Prandtl number,
r	radius, m
R_g	area of gas segment, m^2
R_s	area of gas segment in the stack, m^2
T	temperature, K
t	time, s
u	volumetric flow rate, m^3/s
v	velocity, m/s
β	*isobaric volumetric expansion coefficient,* 1/K
γ	ratio specific heats
f_k, f_v	Rott functions (thermal and viscous)
ε_s	stack heat capacity ratio
μ	dynamic viscosity, m^2/s
ρ	density
ω	angular frequency, Hz

Subscripts

A	amplitude
m	mean value
ref	reference

commercial CFD codes. A lot of attention was devoted in this case to the impact of non-linear effects on the stabilization of a specific amplitude of thermoacoustic oscillations, and the presented results were compared with experimental data [7]. Further works ([15,17]) focused on modelling the operation of a full thermoacoustic engine. In this case, the impact of the resonator curvature on the device basic operating indices was studied. An extension of these findings was the development of the model of a thermoacoustic engine coupled with a thermoacoustic cooler ([16,18]). [19–22] focused on using numerical calculations to determine the impact of the stack geometry on the structure of the flow around its plates and, thereby, on the operating indices of the entire thermoacoustic device. The general purpose Ansys-Fluent code was used in all these investigations.

This paper combines the analytical and numerical approach to an integrated model of the phenomena occurring in the thermoacoustic engine. Both approaches are developed in a manner to find a possibly fast solution on the one hand and – on the other – to compare the obtained results. The geometrical and physical conditions were adjusted to the thermoacoustic phenomena. However, some simplifications were introduced to ensure a better agreement between the two models. So far, this detailed comparison of these 2 models has not been presented in literature. Therefore, the obtained results prove useful in terms of validation and appropriate construction of the model of the thermoacoustic engine. All numerical calculations presented in this paper were performed using the Ansys-CFX code.

2. Thermoacoustic engine

The thermoacoustic engine illustrated in Fig. 1 is made of a resonator, opened to the environment on its one side, a stack and two heat exchangers whose function is to create an appropriate thermal gradient. A standing acoustic wave is generated in the device. Its velocity node is at the closed end of the tube and the velocity antinode – at the opened cross section of the engine.

The basic simplification introduced into the thermoacoustic engine model is to neglect the heat exchangers whose task is to deliver and collect a certain amount of heat. The modelling of the heat exchangers is a very complex task both in the numerical and analytical approach [9]. Instead, a certain distribution of temperature is adopted along the stack walls. The overall dimensions of the device agree with the thermoacoustic engine presented in [17]. Then, it makes it also possible to compare results obtained by means of different commercial codes. Moreover, the presented numerical calculations concerned both the detailed and simplified model of the thermoacoustic engine. The latter is closer to the analytical model.

2.1. Numerical model

All numerical calculations were performed using the general purpose Ansys-CFX software package. The numerical analysis was conducted for the geometry presented in Figs. 1 and 2. Two basic computational variants were taken into account. In the first one, the computational area was composed of several stack plates (Detailed Model) and on its upper and lower edge the "no-slip wall" boundary condition was assumed. In the other (Simplified Model), the analysis only concerned a part of the thermoacoustic engine limited to a single element of the stack. In this case, on the upper and lower edge of the computational area the symmetry condition was assumed. This allowed a substantial reduction in the size of the computational domain. A comparison of these two variants made it possible to take account of the impact of friction in the thermoacoustic engine resonator on the amplitude of its oscillations.

Due to the need to perform time-consuming non-stationary calculations, the analysis was narrowed down to the two-dimensional case. The numerical mesh of the full model of the thermoacoustic engine was composed of 200,000 nodes. For all variants under consideration the assumed time step was 10^{-5} s, which corresponded to 165 iterations per a full period of thermoacoustic oscillations. The basic boundary conditions assumed for the analysis are shown in Fig. 2. In this case, for the stack walls a temperature distribution was assumed in the form of a quarter of the sine curve in the range of 300 and 700 K [17].

A high-resolution scheme was applied to balance the convection and diffusion terms; the second-order accuracy Euler scheme was used for integration with respect to time. The standard k–ε turbulence model was considered in all computations. A detailed comparison of the results obtained from the presented numerical models is given in [11].

The process of thermoacoustic oscillation excitation is shown in Fig. 3. The peak-to-peak value of thermoacoustic oscillations for the detailed and simplified thermoacoustic engine models was 16 kPa and 19.6 kPa, respectively. The difference results mainly from the fact that on the upper and lower edge of the computational domain the symmetrical boundary condition was assumed for the simplified model. As a consequence, the impact of friction on the computed amplitude of thermoacoustic oscillations is ignored. Comparing the results obtained from the detailed model of the thermoacoustic engine with the calculations performed in [17], where the commercial Ansys-Fluent code was applied, the obtained pressure amplitude value was by about 5% smaller.

Figs. 4 and 5 present a comparison of the distributions of pressure and acoustic velocity obtained from the detailed and simplified model. Apart from the calculated amplitude of