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Harmonic mechanical excitations of steady convective instabilities: A means to get more uniform heat transfers in mixed convection flows?

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ABSTRACT

In laminar mixed convection flows, steady thermoconvective patterns generate non uniform heat and/or mass transfers at walls that can be detrimental in some industrial processes. For instance the longitudinal thermoconvective patterns of Poiseuille–Rayleigh–Bénard (PRB) flows generate non uniform thin films or coatings when they are present in cold wall horizontal Chemical Vapor Deposition (CVD) reactors. The aim of this paper is to show that, when the basic steady flow is convectively unstable against an unsteady flow regime, introducing small harmonic mechanical excitations in the basic flow may enable to obtain more uniform time averaged heat transfers. More specifically, three-dimensional direct numerical simulations are used to characterize the temperature field and wall heat transfer associated with unsteady wavy convective instabilities of PRB flows that result from harmonic excitations of the longitudinal thermoconvective rolls at channel inlet. A design of experiments is used to build cubic response surfaces of the different quantities analyzed (growth length of the wavy rolls, magnitude of their spanwise oscillations, wall Nusselt number …) on a wide range of the flow parameters. Air PRB flows ($Pr = 0.71$) in channels of aspect ratios equal to Width/Height = 10 and $150 \leqslant$ Length/Height $\leqslant 300$, for Reynolds numbers $100 \leqslant Re \leqslant 300$ and Rayleigh numbers $5000 \leqslant Ra \leqslant 16{,}000$ are considered. Comparisons with experiments are presented and a good agreement is obtained. The optimal conditions to have uniform heat transfers on the horizontal walls of PRB flows correspond to the minimal growth length of the wavy rolls until saturation and the maximum magnitude of their spanwise oscillations. They are approximately obtained for moderate Reynolds number ($Re \approx 150$), high Rayleigh numbers ($Ra \approx 15{,}000$), low excitation frequency and rather high excitation magnitude. A discussion of these results for the applications to CVD in horizontal rectangular reactors at atmospheric pressure is finally proposed.

1. Introduction

A Poiseuille–Rayleigh–Bénard (PRB) flow is a mixed convection flow in a horizontal rectangular channel heated from below and cooled from above. PRB flows are commonly encountered in industrial applications, for example in heat exchangers, during the air cooling of electronic circuit boards or in the rectangular Chemical Vapor Deposition (CVD) reactors used to make thin solid films or coatings on heated substrates from chemical precursors in gaseous phase (see [1,2] for reviews). To optimize these industrial processes, heat and/or mass transfers at walls must be well controlled as well as the thermoconvective instabilities and the flow type (laminar, transitional, turbulent) that develop in the system. Indeed the thermoconvective instabilities can result in non uniform heat and mass transfers, especially when they are steady, and give rise to a degradation of the desired process (see [2–5] for instance in the case of CVD applications). It is then of great interest to characterize the flow patterns in the PRB configuration to enable a good controlling of the magnitude and homogeneity of heat and mass transfers.

In this paper, only laminar PRB flows are studied. The stable basic state is a purely conductive Poiseuille flow. Its successive destabilizations generate many different thermoconvective patterns, depending on the values of the characteristic parameters: the Reynolds, *Re*, Rayleigh, *Ra*, and Prandtl, *Pr*, numbers and the transverse aspect ratio of the channel, $B = W$ (width)/H (height). Thus the stability diagrams of PRB flows present many flow configurations. A few examples, established experimentally, theoretically

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and numerically, for different Prandtl numbers and aspect ratios, can be found in [6–9] in the case of pure fluids. In the case of air PRB flows ($Pr = 0.7$), a complete stability diagram at $B \geqslant 10$ is presented in [10]. It is partially reproduced in Fig. 1 in order to specify the framework of the present study, in the next paragraphs. Very recently, Barletta and Nield [11] have analyzed the consequences of a uniform internal heat source on the thermal instability of the PRB mixed convection. Finally the stability of PRB flows in binary fluids, with and without Soret effect, is studied in [12–15] for instance.

The primary instability made of steady parallel convection rolls oriented in the direction of the mean flow, referred to as longitudinal rolls, is the main flow pattern in all the stability diagrams of PRB flows at different Pr and B values (see Fig. 1). This instability is observed for sufficiently high Reynolds numbers (typically for $Re > O(10)$ in air) and for Rayleigh numbers above a critical value varying between 1708 and 2000 when $B > 2$ [9]. Carrière and Monkewitz [16] showed that this pattern is a convective instability of the basic conductive Poiseuille flow. Mergui et al. [17] and Benderradji et al. [18] demonstrated that these rolls are triggered in real channels of finite transverse aspect ratio just downstream the leading edge of the heated plate and near the vertical walls due to the presence of velocity and temperature boundary layers adjacent to these walls.

Two secondary unsteady instabilities appear at higher Rayleigh numbers, arising from the destabilization of the longitudinal rolls. They are referred to as oscillating instabilities at low Reynolds number ($Re < O(100)$) and wavy instabilities at high Reynolds number ($Re \geqslant O(100)$) (see Fig. 1). These instabilities have been first detected by Clever and Busse [8] through a time linear stability analysis for an infinite fluid layer. In the current paper, we focus on the wavy instability. The time linear stability analysis of longitudinal rolls against wavy rolls has been extended by Kato and Fujimura [20], Xin et al. [19] and Nicolas et al. [10] to channels of finite transverse aspect ratio. Previous experimental [21] and numerical [10,22] studies showed that the wavy pattern can develop in the channel only if a perturbation is imposed and maintained into the longitudinal roll flow, meaning that the wavy rolls result from a convective instability of the longitudinal rolls.

This feature could be of great interest from a practical point of view. Indeed, the idea is to take advantage of the convective nature and unsteadiness of the wavy instability to enhance or weaken and/or homogenize the heat and/or mass transfers in the industrial processes by imposing the most appropriate perturbations/excitations to the flow. A numerical study by Nicolas et al. [2] has already shown that the presence of wavy rolls generated by harmonic mechanical excitations could homogenize the growth rate and the thickness of the deposited thin solid layers in APCVD (Atmospheric Pressure CVD) reactors. Nicolas et al. [10] conducted a numerical study to characterize the saturated wavy roll flows by maintaining a random excitation, a white noise on the transverse velocity components, at the channel inlet. It has been shown that, depending on the Reynolds and Rayleigh numbers and on the excitation amplitude, the spanwise displacement magnitude of the wavy rolls can be large on a large extent of the domain suggesting that this configuration could potentially be interesting to homogenize the heat and mass transfers in CVD reactors. However, in practical situations, a random excitation is almost impossible to implement and a sinusoidal perturbation will be preferred. Thus, the aim of the present study is to better characterize the spatial and temporal development of the wavy instability and of the associated heat transfers, on a wide range of the control parameters, when a harmonic forcing is imposed to the system. The most effective conditions susceptible to homogenize the heat transfer at the channel walls and the main characteristics of the wavy roll flows (their growth length, the magnitude of the spanwise displacement of the oscillations, the wall Nusselt number at saturation, etc.) will be numerically identified.

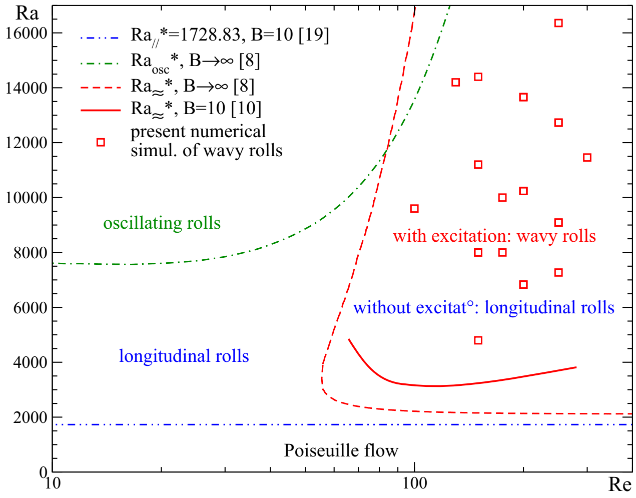

Fig. 1. Primary and secondary marginal stability curves of PRB flows at $Pr = 0.7$ and $B = 10$ or $B \to \infty$ determined by time linear stability analyses in [8,10,19]. $Ra^*_{//}$ is the transition curve between the basic Poiseuille flow and the longitudinal rolls. $Ra^*_{osc}(Re)$ and $Ra^*_{\approx}(Re)$ are the transition curves between the longitudinal rolls and the oscillating and wavy rolls respectively. The simulation points of the wavy roll flows used in the present design of experiments are also indicated.

However, as the wavy roll flows are controlled by six parameters (Re, Ra, Pr, B and the magnitude, A_{exc}, and frequency, f_{exc}, of the harmonic forcing), the complexity of the problem is reduced by setting the values of Pr and B. More precisely, in all this work, $Pr = 0.71$ (air flow) and $B = 10$ to allow experimental and numerical comparisons with the PRB experiments carried out at FAST laboratory [17,21,23,24]. Despite this simplification, the problem remains expensive to solve because one simulation of an unsteady fully-developed three-dimensional (3D) PRB flow requires channels of long streamwise aspect ratios (say $A = L$ (length)$/H \approx 200$), very large grids of more than 10^7 cells or nodes and, as a consequence, high computational resources. The computational cost of such simulations is presented for instance in [25] in the framework of a benchmark exercise on PRB flows, using different numerical methods on parallel or vectorial supercomputers. As the present study aims at analyzing the influence of four parameters (Ra, Re, A_{exc}, f_{exc}) on the wavy roll behavior, on a wide parameter domain, the total computational cost of the study could have been prohibitive. To overcome this difficulty, we decided to apply a design of experiments (DOE) [26]. This technique allows constructing polynomial interpolation surfaces of the studied quantities as a function of all the parameters, on the whole parameter domain, from a limited number of experiments (or simulations in the present case). The accuracy of the interpolation of course depends on the number and repartition of the simulations on the parameter domain and statistical tests are needed to determine it. This aspect will be discussed in the paper.

The paper is organized as follows. First, the mathematical model and the numerical methods are presented in Section 2. The way the design of experiments is built is presented in Section 3. The methodology used and the definitions of the quantities (responses) analyzed to characterize the wavy roll flows are described in Section 4. The results are presented in Section 5. The wavy roll growth lengths until saturation and the magnitude of the most amplified modes at saturation are analyzed and compared with the experiments in Sections 5.1 and 5.2. The magnitude of the spanwise oscillations of the wavy rolls and their spanwise wavelength are studied in Section 5.3. The optimum conditions for uniform time averaged heat transfers on the horizontal plates are determined