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A mathematical model for frost growth and densification on flat surfaces

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ABSTRACT

Many factors including air temperature, humidity, and surface temperature are known to affect frost growth on heat transfer surfaces. In the present study, a new model for frost growth and densification on flat surfaces is presented, accounting for the transport of heat and mass, with special attention to imposing physically realistic boundary conditions. For temperature, a convective boundary condition at the frost-air interface and a fixed cold-surface temperature are used. The water–vapor density at the frost-air interface is not considered as known. Unlike earlier saturation and supersaturation models, the current work adopts a specified heat flux at the cold surface, allowing calculation of the vapor density gradient at the frost-air interface. From the results, it can be shown that the water–vapor at the frost-air interface is supersaturated, as suggested in earlier work. Model predictions of frost thickness and density are in good agreement with experimental data over limited environmental conditions.

1. Introduction

Frosting occurs on heat exchangers in heat pumping and refrigeration applications when the surface temperature of the heat exchanger is below the freezing point of water. The accumulation of frost on the heat transfer surface increases pressure drop (fan power) and decreases heat transfer, because the frost layer blocks the air flow and has a thermal conductivity much lower than that of the heat exchanger material. System efficiency is reduced by frost accumulation, and operation is complicated by the need to defrost the heat exchanger.

Many factors including air temperature, humidity, and cold plate temperature are known to affect frost growth on heat transfer surfaces. The prediction of frost properties is essential to characterizing the performance of heat exchangers under frosting conditions. A number of experimental studies have been reported (see [1–5]); however, frost properties depend on many factors (air temperature, humidity, surface temperature, and surface wettability), making it challenging to conduct sufficient experiments to cover all combinations of factors. Therefore, many analytical or numerical models to predict frost growth have been developed. Most extant mathematical models rely on solving the heat and mass diffusion equations in the frost layer. They can be classified into two categories, based on what boundary conditions are adopted. The first and most common category is the saturation models in which the water vapor is assumed to be saturated at the cold surface and at the frost-air interface (e.g., Hermes et al. [6] and Lee et al. [7]). Lee et al. [7] developed a saturation model for frost formation on cold flat surfaces that was based on the assumption that the amount of water vapor absorbed into the frost layer is proportional to the vapor density in that layer [6]. With this simplifying assumption, an analytical solution for the water vapor density and temperature distribution in the frost layer was obtained, assuming the process is quasi-steady. From the model, it was concluded that frost thickness and frost-air interface temperature increase with increasing air velocity and air relative humidity.

Lee et al. [8] proposed a modification to the earlier model, in which the boundary layer equations with appropriate boundary conditions were solved for specified air temperature, humidity, and velocity. Yang and Lee suggested yet another modification to the previous model by including an experimental correlation for frost density [9]. The frost density was assumed to be a function of all frost growth parameters including air velocity, temperature, and humidity, and surface and frost-air interface temperatures. The main advantage of this approach is that it does not require the knowledge of an initial frost density. However, the proposed empirical correlations are valid only for a limited range of experimental conditions. Moreover, the assumption of saturation conditions at the frost-air interface has been called into question [10–12].

More recently, Na and Webb proved that water vapor at the frost-air interface is not saturated but supersaturated, as a laminar

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Nomenclature

BF	blowing factor, $BF = \ln(1+B)/B$ (–)
B	mass transfer driving force $B = (m_s - m_w)/(m_w - 1)$ (–)
D	mass diffusivity of water in air (m²/s)
h	heat transfer coefficient (W/m² K)
h_m	mass transfer coefficient (m/s)
h_{sg}	latent heat of sublimation (J/kg)
k	thermal conductivity (W/m K)
L	plate length (m)
Le	Lewis number, α/D (–)
m''	mass flux (kg/m² s)
Nu	Nusselt number, $Nu = hL/k_a$ (–)
Pr	Prandtl number, ν/α (–)
q''	heat flux (W/m²)
Sh	Schmidt number, ν/D (–)
T	temperature (K)
U	Darcy velocity (m/s)

Greek letters

α	thermal diffusivity (m²/s)
δ	frost thickness (m)
ε	mass transfer conductance (S^{-1})
ν	kinematic viscosity (m²/s)
ϕ	porosity (–)
ρ	density (kg/m³)
τ	tortuosity (–)
ω	relative humidity (–)

Subscripts

a	air
fst	frost
lat	latent
ρ	density
δ	thickness
t	total
s	surface
sat	saturated
$sens$	sensible
v	vapor

boundary layer analysis suggests [10–12]. However, their analysis assumed the temperature and concentration profiles in the frost layer were linear. No support for this assumption was provided, and it is patently incorrect. Unlike earlier models, their supersaturation model for frost growth on flat plates accounted for the variation of frost density along the frost thickness. The gradient of the frost density at the frost-air interface was taken to be zero to allow calculation of the density of the fresh frost that deposited at the surface at each time step. The degree of supersaturation was predicted from experimental data and the proposed correlation is valid only for $T_\infty - T_{fst,s}$ ranging from 14 to 20 °C.

Lee and Ro suggested correlations to determine the degree of supersaturation and the diffusion factor [13]. The degree of supersaturation determines the total mass flux of water–vapor from moist air to the frost layer, while the diffusion factor determines the contribution of the total mass flux to the densification of the frost layer. In their study, different expressions for the diffusion coefficient were explored to determine which correlation best represented experimental data.

Although the argument for supersaturation put forward by Na and Webb [10] is not flawless, it is compelling. The cogency of their argument notwithstanding, the saturation assumption is still being used (e.g., [7,9]), because of the absence of a general method or widely applicable correlation to find the degree of supersaturation at the frost-air interface. Na and Webb correlated the degree of supersaturation as a function of the vapor pressure of the surrounding air and the saturation vapor pressure at the frost-air interface temperature. However, Kandula suggested that the degree of supersaturation degree on other factors, such as the surface wettability, and thus recommended the saturation boundary condition as more convenient to use [14].

In the present study, a new mathematical model is developed, with improved boundary conditions addressing the issue of supersaturation. The new model is used to predict frost growth on flat plates.

2. Frost growth and densification

2.1. Frost growth mechanisms

The mechanisms involved in the growth and densification of a frost layer on a cold surface are shown schematically in Fig. 1.

As moist air passes above a cold surface having a temperature below the freezing point of water, frost start forming on the surface. The early growth has been characterized as a "crystal growth period;" it is followed by "frost layer growth period," during which a frost layer grows by ablimation [5]. Frost growth involves simultaneous heat and mass transfer driven by the temperature and water mole fraction differences between the frost layer and the surrounding air. The total heat flux can be divided into sensible and latent components. The latent heat flux is directly related to the mass flux of water vapor that deposits on the frost layer, a portion of which deposits at the frost-air interface and contributes to layer growth (thickening), and a portion of which is driven into the layer by the gradient in the partial density of water vapor at the frost-air interface and increases the frost density.

2.2. Mathematical formulation

Assuming that the frost density and effective thermal conductivity vary within the frost layer (in the z-direction), a control volume analysis is performed in order to relate the frost growth to the air properties and cold surface temperature.

The total heat flux transferred from air to the frost layer can be divided into a sensible and a latent heat flux as follows:

$$q''_a = q''_{lat,t} + q''_{sens} \quad (1)$$

$$q''_{sens} = h(T_a - T_{fst,s}) \quad (2)$$

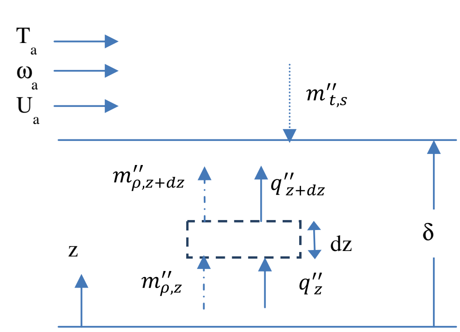

Fig. 1. Schematic of the frost layer.