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Effective thermal conductivity of composites

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ABSTRACT

The paper employs a theoretical model to evaluate the thermal conductivity of composites under two thermal assumptions which allow to solve the heat conduction equation. The composite is made of a ceramic-silica matrix and a fiber reinforcement. Three different materials are investigated as reinforcement, i.e. asbestos, steel and copper. The theoretical effective thermal conductivity is calculated along the three directions for a non-consumed composite and during its consumption. Numerical solutions of heat conduction in the composite are carried out. The anisotropy degree of the composite is investigated for the composite during its consumption. The anisotropic efficiency of the composite is defined as ratio between the anisotropy degree of the composite and the potential anisotropy of the two materials, i.e. the ratio between the thermal conductivity of the fiber and the matrix. The theoretical model allows to evaluate, under the two thermal assumptions, the anisotropic efficiency of the composites which decreases with the increase of the potential anisotropy reaching a minimum, which is only dependent on the geometry of the composite.

1. Introduction

The thermal behavior of composites, made of two materials with different thermal properties is important in several applications as thermal protections, heat shields, heat guides and in cases where anisotropy is requested. Several researchers have investigated composites with different approaches.

Torquato and Kim [1] computed the effective properties of heterogeneous media with an efficient simulation technique. Havis et al. [2] used the finite difference method to simulate the thermal behavior of fiber reinforced composites, with the conclusion that these materials can be considered homogeneous and thermally anisotropic. Graham and McDowell [3] evaluated the effective thermal conductivity of a composite made of two different materials by means numerical simulations of the microscopic structure of the composite. Islam and Pramila [4] carried out a numerical analysis of the transverse thermal conductivity of composites with imperfect interfaces. Dasgupta and Agarwal [5] developed a 2-D thermal method based on series/parallel arrangement. Gu and Tao [6] studied a periodic composite with contact resistance with the conclusion that the contact resistance can change the effective conductivity dramatically. James and Harrison [7] evaluated the effective thermal conductivity of composites, made of two materials, simulating numerically the microscopic structure of the composite for different orientations of the anisotropic materials. Torquato and Rintoul [8] investigated the effect of the interface on the properties of composite media. Wang [9] determined the thermal conductivity of a material containing a layer or a staggered array of thin strips. Shim et al. [10] investigated the thermal conductivity of carbon fiber-reinforced composites with different fiber cross-section types, with emphasis on anisotropy and thermal diffusivity factors. Lim [11] evaluated the thermal conductivity of unidirectional continuous fiber-composites, particulate composites and laminate sheets. Terron and Sanchez Lavega [12] and Salazar et al. [13] determined the effective thermal diffusivity of fiber-reinforced composites. Skorokhod [14] studied multi-layer composites, made of ceramic and metallic sheets, calculating the effective thermal properties, with the conclusion that the composite material is thermally anisotropic. Felske [15] studied the thermal conductivity of carbon fiber-reinforced composites with different fiber cross-section types. Goyhénèche and Cosculluela [16] presented multi-scale modeling of the effective thermal conductivity tensor of a stratified composite material made of carbon fibers, phenolic resin, and carbon loads. Al-Nassar [17] deduced empirical formula to predict the thermal conductivity of a composite material with estimated air void volume percent and the relative 3-D problem was modeled using a finite element analysis. Yuan and Luo [18] studied composites of silicone matrix, filled with phosphor and used in light emitting diodes packaging, for predicting the thermal conductivity and experiments were carried out to validate the model.

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Nomenclature

Latin

a	thickness in x, y directions
b	thickness in x, y directions
c	specific heat
k	thermal conductivity
L	length
q	heat flux
$\dot{Q}$	heat transfer
s	generic thickness in z direction
s_1, s_3	matrix thickness
s_2	fiber thickness
T	temperature

Greek

Δ	finite difference
ξ	anisotropy degree
ρ	composite density
ϕ	reinforcement volume fraction
χ	anisotropic efficiency
δ	potential anisotropy

Subscripts

f	fiber
m	matrix
max	maximum
min	minimum
x, y, z	directions
i, j, k	versors

A thermal protection composite was studied theoretically and numerically in [19] during thermal degradation. The thermal behavior of a composite, made of silica matrix and fibrous asbestos reinforcement and used as heat shield, was studied theoretically and numerically during the thermal ablation of a re-entry mission at high Mach number [20]. A composite material made of two plane layers, where one material is silica and the other one is asbestos, or steel or copper has been investigated theoretically in [21,22].

This paper employs a theoretical model for the thermal conductivity of composites, subjected to consumption, investigating the thermal anisotropy behavior. The theoretical approach employs the cubic cell model with two thermal assumptions, parallel isothermal lines and heat flux lines, originally proposed in [23,24], later on extended to non-saturated soils [25], compared to experimental data [26–29], applied to space problems [30–33] and compared to other theoretical models [34–40].

2. Theoretical models

The elementary cubic cell of the material is presented, in three-dimensions, in Fig. 1, with the fiber reinforcement in the middle and the ceramic–matrix around. The upper and lower layers of the matrix have thicknesses s_1 and s_3, while the intermediate layer, containing fiber and matrix, has thickness s_2.

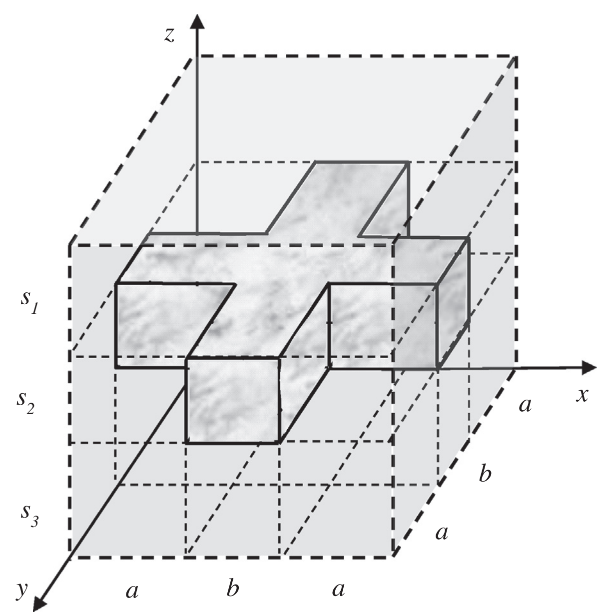

Fig. 1. Elementary cubic cell.

The reinforcement volume fraction is

$$\phi = \frac{s_2 \cdot \left[(2a+b)^2 - 4a^2\right]}{(s_1+s_2+s_3)\cdot(2a+b)^2} \quad (1)$$

The heat conduction equation in three dimensions for a non-isotropic material is

$$\vec{q} = \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = -[k]\cdot\vec{\nabla}T = -\begin{pmatrix} k_x & k_{xy} & k_{xz} \\ k_{yx} & k_y & k_{yz} \\ k_{zx} & k_{zy} & k_z \end{pmatrix}\cdot\vec{\nabla}T \quad (2)$$

The symmetry conditions of the elementary cubic cell of Fig. 1, $k_{ij} = k_{ji}\,(i{\neq}j)$, allow to assume $k_x = k_y$, $k_{xy} = k_{yx}$, $k_{zy} = k_{yz}$ and $k_{xz} = k_{zx}$, in agreement to [41], and the thermal conductivity tensor can be reduced to

$$\begin{pmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{pmatrix} \quad (3)$$

The heat conduction equation can be solved under the two thermal assumptions of parallel isotherms, or parallel isothermal lines (PIL), and parallel heat fluxes, or parallel flux lines (PFL), to define the thermal conductivity in the generic i direction

$$k_i = \frac{\dot{Q}_i \cdot L_i}{\Delta T_i \cdot A_i} \quad (4)$$

2.1. Parallel isothermal lines (PIL)

The assumption of parallel isothermal lines, PIL, means that the thermal conductivity in the transversal direction, i.e. on the isothermal plane, is infinitely high. The effective thermal conductivities in x and z directions, indicated as k_x and k_z, are calculated with the assumption that the temperature at $z = 0$ or $x = 0$ is T_1, at $z = s_3$ or $x = a$ is T_2, at $z = s_3 + s_2$ or $x = a + b$ is T_3 and at $z = s_3 + s_2 + s_1$ or $x = 2a + b$ is T_4.

The total heat transfer in the x direction is the same throughout each cross section, in steady state, and is

$$\dot{Q}_x = k_x \cdot A_x \cdot \frac{\Delta T_x}{\Delta x} = k_x \cdot (2a+b)\cdot(s_1+s_2+s_3)\cdot\frac{T_1 - T_4}{(2a+b)} \quad (5)$$