International Journal of Heat and Mass Transfer 77 (2014) 843–851

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

# Weakly nonlinear oscillatory convection in a viscoelastic fluid saturating porous medium under temperature modulation

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ARTICLE INFO

*Article history:*
Received 14 January 2014
Received in revised form 17 May 2014
Accepted 19 May 2014

*Keywords:*
Temperature modulation
Supercritical flow
Viscoelastic fluids
Nonlinear stability

ABSTRACT

A study of thermal instability driven by buoyancy force is carried out in an initially quiescent infinitely extended horizontal porous medium saturated with viscoelastic fluid. Modified Darcy's law is used to explain characteristics of fluid motion. The time-periodic temperature on the boundaries has been considered and its effect on the system has been investigated. A weak nonlinear stability analysis has been performed for the oscillatory mode of convection, and heat transport in terms of the Nusselt number, which is governed by the complex non-autonomous Ginzburg–Landau equation, is calculated. The influence of relaxation and retardation times of viscoelastic fluid on heat transfer has been discussed. Further, the study establishes that the heat transport can be controlled effectively by a mechanism that is external to the system. Finally, it is found that supercritical flow advances the onset of convection hence increases heat transfer.



## 1. Introduction

Thermal convection in porous media is a subject of considerable interest in contemporary fluid flow and heat transfer research. Its importance stems from a wide range of occurrences in industrial applications and geological systems. From a purely scientific point of view, porous convection is also of great interest because it is one of the simplest systems exhibiting nonlinear instability. The problem of convective instability of a horizontal fluid saturated porous medium has been investigated extensively and is well documented by Ingham and Pop [1], Nield and Bejan [2] and Vafai [3]. The Rayleigh–Bénard instability for a Newtonian fluid in a porous medium was first investigated by Horton and Rogers [4] and later by Lapwood [5]. There are many situations of practical importance in which temperature gradient is a function of both space and time. This temperature gradient can be determined by solving the energy equation with suitable time-dependent thermal boundary conditions, and can be used as an effective mechanism to control the convective flow. Venezian [6] was the first to perform a linear stability analysis of Rayleigh–Bénard convection for the case of small-amplitude temperature modulation. Using perturbation method and considering free–free surfaces, he calculated the shift in the critical value of the Rayleigh number and showed that the system can be stabilized or destabilized by suitably tuning the modulation frequency. Rudraiah and Malashetty [7] investigated the stability of the fluid-saturated sparsely packed porous layer subject to time-periodic boundary temperature using the Brinkman model. They recovered the viscous flow results of Venezian [6], as a special case when the value of the porous parameter tends to zero.

The effect of time-dependent wall temperature on the onset of convection in a fluid saturated porous medium has been studied by Caltagirone [8]. He used the Darcy model for the momentum equation and employed linear theory to study theoretically the stability of the system by the Galerkin technique. Chhuon and Caltagirone [9] studied the stability of a fluid saturated porous layer imposing time-periodic temperature modulation on the boundary, with a non-zero mean value. They performed experiments and compared their results with those obtained when Floquet theory was used. Employing Forchheimer flow model Malashetty and Wadi [10] studied the stability of a Boussinesq fluid-saturated horizontal porous layer, heated from below, using time-dependent wall temperature. It was found that the system was most stable when the boundary temperature was modulated out of phase. Also, they found that the low-frequency thermal modulation could have a significant effect on the stability of the system. Employing

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.05.037
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**Nomenclature**

*Latin symbols*

| | |
|---|---|
| $\mathbb{A}_1(s)$ | amplitude of convection |
| $a$ | wave number |
| $\delta$ | amplitude of temperature modulation |
| $d$ | depth of the fluid layer |
| $\mathbf{g}$ | acceleration due to gravity |
| $Nu$ | Nusselt number |
| $p$ | reduced pressure |
| $Ra_D$ | thermal Rayleigh–Darcy number, $Ra_D = \frac{\beta_T g \Delta T d K}{\nu \kappa_T}$ |
| $R_{0c}$ | critical Rayleigh–Darcy number |
| $T$ | temperature |
| $\Delta T$ | temperature difference across the porous layer |
| $t$ | time |
| $(x, z)$ | horizontal and vertical co-ordinates |

*Greek symbols*

| | |
|---|---|
| $\alpha_T$ | coefficient of thermal expansion |
| $\chi$ | perturbation parameter |
| $\kappa_T$ | effective thermal diffusivity |
| $K$ | permeability |
| $\gamma$ | heat capacity ratio $\gamma = \frac{(\rho c)_m}{(\rho c)_f}$ |
| $\Omega$ | frequency of modulation |
| $\omega$ | dimensionless oscillatory frequency |
| $\bar{\lambda}$ | stress relaxation time |
| $\bar{\varepsilon}$ | strain retardation time |
| $\mu$ | dynamic viscosity of the fluid |
| $\phi$ | porosity |
| $\nu$ | kinematic viscosity, $\left(\frac{\mu}{\rho_0}\right)$ |
| $\rho$ | fluid density |
| $\psi$ | stream function |
| $s$ | slow time scale |
| $\tau$ | fast time scale |
| $\phi$ | phase angle |

*Other symbols*

| | |
|---|---|
| $\nabla^2$ | $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ |

*Subscripts*

| | |
|---|---|
| $b$ | basic state |
| $c$ | critical |
| 0 | reference value |

*Superscripts*

| | |
|---|---|
| $'$ | perturbed quantity |
| $*$ | dimensionless quantity |

Brinkman flow model Malashetty and Basavaraj [11] investigated a linear stability analysis in an anisotropic porous medium, and found that it was possible to advance or delay the onset of convection by time-periodic modulation of the wall temperature, and to advance it by gravity modulation. It was also found that the small anisotropic parameter had a strong influence on the stability of the system. The same authors analyzed the effect of temperature modulation on double diffusive convection in anisotropic porous medium [12], and found that it was possible to advance or delay the onset of double diffusive convection by proper tuning of the frequency of modulation of the wall temperature. The linear stability of thermal convection in a rotating horizontal layer of fluid-saturated porous medium, confined between two rigid boundaries, was studied for temperature modulation, using Brinkman's model by Bhadauria [13]. It was found that both, rotation and permeability were having stabilizing influence on the onset of thermal instability. Further, he found that it was possible to advance or delay the onset of convection by proper tuning of the frequency of modulation of the walls' temperature. The effect of temperature modulation on the onset of double diffusive convection in a sparsely packed porous medium is studied by Bhadauria [14] making a linear stability analysis, and using Brinkman–Forchheimer extended Darcy model. He investigated the effect of permeability and thermal modulation on the onset of double diffusive convection using Galerkin method and Floquet theory, and compared the results of Brinkman and Darcy models.

Bhadauria and Sherani [15] studied the effect of temperature modulation on the onset of thermal convection in an electrically conducting-fluid-saturated porous medium, and investigated the influences of magnetic field and Vadasz number on the system. They found that the effect of temperature modulation on the onset of convection is to advance or delay the convection, depending on the proper tuning of the frequency of modulation. Thermal instability in an electrically conducting two component fluid-saturated-porous medium was investigated by Bhadauria and Srivastava [16], considering temperature modulation of the boundaries. The porous medium was confined between two horizontal surfaces, and subjected to a vertical magnetic field; flow in the porous medium was characterized by Brinkman–Darcy model. By comparing the results of Brinkman and Darcy models, they found that there is very good agreements between two models when the Datcy number $Da$ was small. Siddheshwar et al. [17] studied the effect of both temperature and gravity modulation of binary convection in a porous medium, considering nonlinear theory. Using non-autonomous Ginzburg–Landau equation, they investigated heat and mass transfer for stationary convection. Bhadauria et al. [18] studied the effect of time-periodic temperature/gravity modulation on thermal instability in a fluid-saturated rotating porous layer, by performing a weakly nonlinear stability analysis. Using stationary mode of convection, the individual effects of temperature and gravity modulation on heat transport was investigated. Siddheshwar et al. [19] analyzed magneto-convection in an electrically conducting fluid layer under both temperature and gravity modulation, considering a weak nonlinear theory. Bhadauria et al. [20] investigated internal heating and thermal modulation effects, Bhadauria and Kiran [21], thermal modulation in an anisotropic porous medium, performing weakly nonlinear theory for stationary mode of convection. The influence of synchronous and asynchronous boundary temperature modulations on Darcy Bénard convection was studied by Siddheshwar et al. [22], using a new way of approach to determine the effect of frequency and phase angle on average value of Nusselt number.

The study of non-Newtonian fluid flow in a porous medium is of considerable importance in various areas in science, engineering and technology, such as material processing, petroleum, chemical and nuclear industries, bioengineering, geophysics and reservoir engineering etc. The performance of a reservoir depends to a large extent upon the physical nature of crude oil present in the reservoir. The light crude is of Newtonian type and is studied extensively using the Darcy equation. On the other hand, the heavy crude oil is of non-Newtonian type and a study of such fluids is based on a generalized Darcy equation, which takes into account