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An incompressible smoothed particle hydrodynamics method for natural/mixed convection in a non-Darcy anisotropic porous medium

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ABSTRACT

In this paper, the non-Darcy model for natural/mixed convection and heat transfer in a cavity saturated with anisotropic porous media has been investigated using an incompressible smoothed particle hydrodynamics (ISPH) method. In the ISPH algorithm, a semi-implicit velocity correction procedure has been used and the pressure is obtained by solving pressure Poisson equation. The unsteady natural/mixed convection in non-Darcy porous cavities is examined by our extended ISPH method and Finite Volume Method. The results are presented with flow configurations, isotherms and average Nusselt numbers for different Darcy numbers from 10^{-4} to 10^{-2}, porosity values from 0.4 to 0.9, permeability ratio 0.1–10, inclination angle of permeability 0–90° and Reynolds/Rayleigh numbers. The flow pattern and rate of heat transfer inside the cavity are affected by these parameters. The results demonstrate the effects of the parameters such as Darcy number, porosity, permeability ratio and inclination angle in both of the heat transfer rate and the flow regime. The results from this investigation are well validated and have favorable comparisons with previously published results.

1. Introduction

Darcy's law is a phenomenologically derived constitutive equation that describes the flow of a fluid through a porous medium. One application of Darcy's law is to water flow through an aquifer; Darcy's law along with the equation of conservation of mass are equivalent to the groundwater flow equation, one of the basic relationships of hydrogeology. Darcy's law is also used to describe oil, water, and gas flows through petroleum reservoirs. Also, one of the porous media concepts is the permeability. This concept can be used in determining the flow characteristics of hydrocarbons in oil and gas reservoirs, and of groundwater in aquifers. On the other hand, the natural convection in an anisotropic porous medium is an important area of research due to its wide range of applications including thermal insulation, flow in mushy region of a solidifying alloy [1] and flow past heat exchanger tubes [2]. The non-Darcy effects on natural convection in porous media have also received significant attention as a result of the experiments conducted with several combinations of solids and fluids. These experiments covered wide ranges of governing parameters that indicate that the experimental data for systems other than glass water at low Rayleigh numbers do not agree with the theoretical predictions based on the Darcy flow model. This divergence in the heat transfer results has been reviewed in detail in Cheng [3] and Prasad et al. [4], among others. Thus, extensive efforts are being made to include the inertia and viscous diffusion terms in the flow equations and to examine their effects in order to develop a reasonably accurate mathematical model for convective transport in porous media. Detailed accounts of the research into non-Darcy convection have been reported in Tien and Hong [5], Cheng [6], Prasad et al. [7], and Kiadias and Prasad [8]. Nield and Bejan [9] provided an excellent summary of the subject regarding porous media models. The numerical studies of the natural convection flow in anisotropic porous media were conducted by use of Brinkman equation [10] or Brinkman–Forchheimer equation with permeability tensor [11]. They demonstrated that their formulations were accurate in predicting the flow and heat transfer for various inclinations of the principal permeability direction, permeability ratios, and Darcy numbers. The natural convective flow and heat transfer in a fluid saturated anisotropic porous medium have been investigated using the generalized non-Darcy model as Nthiarasu et al. [12].

In recent years, the smoothed particle hydrodynamics (SPH) method had been applied into compressible and incompressible viscous fluid flow problems [13,14]. The SPH was originally developed in compressible flow, and then some special treatment was required to satisfy the incompressible condition. One approach is

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Nomenclature

C	Forchheimer coefficient
C^*	ratio of Forchheimer coefficient
C_p	specific heat
Da	Darcy number
d_0	particle size
F	Forchheimer constant
F^*	ratio of Forchheimer constant
g	gravitational acceleration vector
H	cavity height
K	permeability
k	thermal conductivity
K^*	permeability ratio
k^*	thermal conductivity ratio
L	cavity width
Nu	Nusselt number
P	pressure
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynolds number
T'	temperature
t	time
$\mathbf{V}$	velocity vector
U,V	dimensionless velocity
x,y	Cartesian coordinates
X,Y	dimensionless coordinates

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
ϵ	porosity
μ	viscosity
υ	kinematic viscosity
ρ	density
τ	dimensionless time
θ	inclination of principal axes

to run the simulations in the quasi-incompressible limit, that is, by selecting the smallest possible speed of sound which still gives a very low Mach number ensuring density fluctuations [13,14]. This method is known as the weakly compressible smooth particle hydrodynamics (WCSPH). In the WCSPH, the artificial viscosity, which is originally developed by Monaghan and Pongracic [15], has been widely used not only for the energy dissipation but also for preventing unphysical penetration of particles.

A proposal for developing an incompressible SPH (ISPH) model has been introduced, which pressure is implicitly calculated by solving a discretized pressure Poisson equation at every time step [16–30]). Cummins and Rudman [16] introduced a new formulation for enforcing incompressibility in smoothed particle hydrodynamics (SPH). The method uses a fractional step with the velocity field integrated forward in time without enforcing incompressibility. The resulting intermediate velocity field is then projected onto a divergence-free space by solving a pressure Poisson equation derived from an approximate pressure projection. Lee et al. [17] presented comparisons of a semi-implicit and truly ISPH algorithm with the classical WCSPH method, showing how the ISPH model could resolve some problems encountered in incompressible flow simulation by using WCSPH. Khayyer et al. [18,19] proposed a corrected incompressible SPH method (CISPH) based on a variation approach to ensure the angular momentum conservation of ISPH formulations to improve the pressure distribution by improvement of momentum conservation and the second improvement is achieved by deriving and employing a higher order source term based on a more accurate differentiation. Hu and Adams [20–22] introduced angular-momentum conservative smoothed particle dynamics for incompressible viscous flows and they adapted ISPH method for multi-phase flow. Asai et al. [26] introduced the stabilized incompressible SPH method by relaxing the density invariance condition. Aly et al. [27–29] applied the stabilized incompressible SPH method to simulate multi-fluid problems, fluid–structure interaction and fluid–soil–structure interactions.

Numerical modeling of transient natural convection by using SPH method has also been investigated. Chaniotis et al. [30] proposed a remeshing algorithm based on weakly compressible flow approach and performed a comprehensive study for non-isothermal flows. Remeshing procedure was tested for various benchmark problems for fluid and energy transport, which include1-D shock-tube problem, 2-D Taylor–Green flow, 2-D double shear layer, lid-driven flow in a square cavity, natural convection in a differentially heated cavity and mixed convection in a driven cavity. From the results, it was found that remeshing improves the accuracy of simulations since uniform particle spacing was conserved in each time step. Moreover, Chaniotis et al. [30] used remeshing procedure in the following studies including effect of jet pulsation on heat transfer and flow characteristics of single and double jet impingement on a heated surface, laminar chemically reacting flow and interfacial flows. SPH simulation of flow and energy transport using SPH was performed by Szewc et al. [31]. Natural convection in a square cavity problem with a Boussinesq and a non-Boussinesq formulation was studied. They introduced a new variant of the smoothed particle hydrodynamics (SPH) simulations of the natural convection phenomena. Danis et al. [32] modeled the transient and laminar natural convection in a square cavity using SPH method with a discretization tool on uniform Eulerian grids.

The objective of this study is to present a generalized porous medium model based on the ISPH method for natural/mixed convection and heat transfer in a cavity saturated with anisotropic porous media. It is important to have a suitable and simple solution procedure to solve the transient porous medium equations using ISPH method. In this paper, we describe the implementation of the projection method procedure for a more general hydrodynamically and thermally anisotropic porous medium. A semi-implicit time integration scheme is applied for the anisotropic porous media. Moreover, the evaluation of pressure is stabilized as Asai et al. [26] by including both of velocity divergence and density invariance into pressure Poisson equation.

The unsteady natural/mixed convection in non-Darcy porous cavities is examined by our extended ISPH method and Finite Volume Method. The obtained results by Finite Volume Method are introduced as validation tests. The results are presented with flow configurations, isotherms and average Nusselt numbers for different Darcy numbers from 10^{-4} to 10^{-2}, porosity values from 0.4 to 0.9, permeability ratio 0.1–10, inclination angle of permeability 0–90° and Reynolds/Rayleigh numbers. The flow pattern and rate of heat transfer inside the cavity are affected by these parameters. The results demonstrate the effects of the parameters such as Darcy number, porosity, permeability ratio and inclination angle in both of the heat transfer rate and the flow regime. The results from this investigation are well validated and have favorable comparisons with previously published results.