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Non-Newtonian two-phase stratified flow with curved interface through horizontal and inclined pipes

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ABSTRACT

A model which predicts liquid holdup (ε_{TPL}), pressure gradient and frictional multiplier (ϕ_L) for non-Newtonian liquid–gas flow with different interfacial shapes is presented. The interfacial shape is calculated by solving the Young–Laplace equation, while the non-Newtonian liquid is a power-law shear-thinning liquid. The effects of interfacial tension and interfacial shape are included in the model. The results show that the interface shape can be assumed as an arc shape when Bond number is lower than 10. Furthermore the model can predict liquid holdup, two-phase pressure gradient and frictional multiplier for uniform stratified flows. In addition, the result indicate that the interfacial effects on the holdup and pressure difference between two phases are significant, especially for small liquid holdups.

1. Introduction

Stratified two-phase flow, which is considered a fundamental flow configuration in horizontal and inclined two-phase systems of a finite density differential, has been of long-standing interest from both the practical and the theoretical points of view. In practice, many pipeline systems were designed to operate in the stratified flow region. From a theoretical point of view, a stratified two-phase flow is a basic flow pattern and is often used as a starting point in modeling flow patterns transitions. In many practical stratified gas–liquid two-phase flows, the gas phase may be turbulent and inter entrainment and aeration between two phases may occur at high velocities. At the same time, the liquid phase is laminar and the interface still represents an important role. Thus, there have been a variety of studies of stratified two-phase flow for both horizontal and inclined pipe lines [1–4].

The flow geometry of stratified flow in circular conduits is very complex. In order to simplify the model, most of researchers assumed that the interface between the phases is planar [1–6]. However, Russell and Charles [5], Ng et al. [7] and Charles and Redberger [8] indicated that the assumption of planar interface influenced the accuracy in predicting the axial pressure drop and holdup of liquid. Thus accuracy is poor when the interface is assumed as planar. The effect of surface tension and gravity are characterized by the Bond number, *Bo*, given by

$$Bo = \frac{\Delta \rho g R^2}{\sigma} \tag{1}$$

where $\Delta\rho$ is the density difference between two fluids, g the gravitational constant, R the radius of the pipe and σ is the interfacial tension. The larger the Bond number, the more closely the interface approaches a planar surface.

Traditionally, the consideration of interface curvature is related to capillary and small scale systems, where the effect of surface tension becomes comparable with gravity. On the other hand, stratified flow pattern, particularly when the viscosity ratio is high, the interface curvature and its influence on wetted areas have a crucial effect on the flow pressure drop [5,8]. The interfacial shape depends on the contact angle, the Bond number and the volume fraction of two fluids. Many attempts have been made to calculate the interfacial shape using various theories. Soleimani [9] determined the interfacial shape of a liquid–liquid system by solving the Yong–Laplace equation numerically. Brauner et al. [10] predicated the interface curvature using energy considerations. Gorelik [11,12] and Ng et al. [13,14] obtained exact analytical solution for the interface shape by solving the Young–Laplace equation with various contact angle. They found that the circular arc approximation provides a very good model for all values of the dimensionless parameters. The flow behaviors of stratified flow with curved interface can also be found in Refs. [5,7,15–19]. The results indicate that

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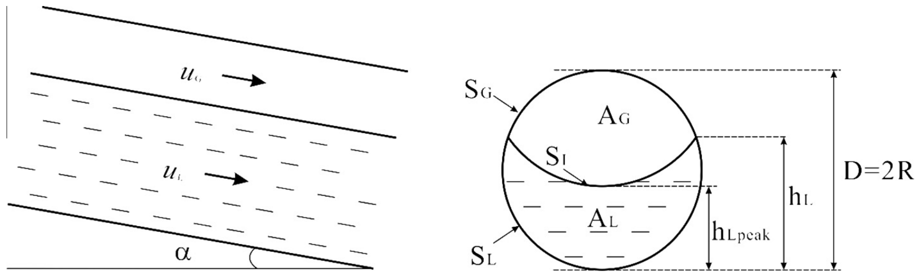

Fig. 1. Stratified non-Newtonian liquid–gas flow with curved interface.

the curved interface significantly affect the local and integral two-phase flow characteristics and transition between different flow patterns [20].

Most of previous works focused on Newtonian liquid–liquid two-phase flow. Compared with Newtonian fluids, the flow behavior of non-Newtonian fluids are more difficult to predict because the shear stress of non-Newtonian fluids changes as the shear rate is varied [21]. Previous research efforts on the non-Newtonian gas–liquid two-phase flows [22–27] assumed that the interface is planar, thus the influence of interface shape was ignored. In applications, the diameter of the pipe varies from meters to centimeters, and the effect of surface tension becomes significant as the diameter of the pipe decreases.

Here, we develop a theoretical model to predict two-phase pressure gradient, holdup of liquid and frictional multiplier of stratified non-Newtonian gas–liquid flow under horizontal or incline conditions. The stratified flow are schematically shown in Fig. 1. In this figure, α means the angle of inclination, $0 \leqslant \alpha \leqslant \frac{\pi}{2}$. This model includes two special conditions: $\alpha = 0$ (the pipe is horizontal) and $\alpha = \frac{\pi}{2}$ (the pipe is vertical).

2. Mathematical model

2.1. Interfacial shape

The motion and change of the interface is defined using the following equation

$$F(\mathbf{r}, t) = 0 \tag{2}$$

where $\mathbf{r}$ is the position vector, and t is time. The unit normal vector $\mathbf{n}$ to the interface is described as [28]

$$\mathbf{n} = \pm \frac{\nabla F}{|\nabla F|} \tag{3}$$

where the sign of "+" means the normal $\mathbf{n}$ directed outward and "−" is for $\mathbf{n}$ directed inward. The stress balance on both sides of the interface can be described as [29]

$$(\mathbf{T}_1 \cdot \mathbf{n} - \mathbf{T}_2 \cdot \mathbf{n}) + \nabla_s \sigma - \sigma \mathbf{n}(\nabla_s \cdot \mathbf{n}) = 0 \tag{4}$$

where $\mathbf{T}$ is the viscous stress, ∇_s is the surface gradient operator, σ is the surface tension. For incompressible fluids, $\mathbf{T}$ can be described as [28]

$$T_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

where p is pressure. ∇_s can be described as [28]

$$\nabla_s = \frac{\partial}{\partial x_i}\mathbf{i} + \frac{\partial}{\partial x_j}\mathbf{j} \tag{6}$$

The stress balance on the both side of the interface along normal direction can be described as

$$\mathbf{n}_1 \cdot \mathbf{T}_1 - \mathbf{n}_2 \cdot \mathbf{T}_2 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\sigma \mathbf{n}_1 \tag{7}$$

where R_1 and R_2 are the principle radius of curvature. For the fully developed flow, the interface in the flow direction does not change, and hence it can be described as

$$z = \eta(y) \tag{8}$$

Substituting Eq. (8) into Eq. (2), the governing equation about interfacial shape becomes

$$F(\mathbf{r}, t) = z - \eta(y) = F(y, z) \tag{9}$$

The surface shape is determined in Eq. (8), then the differential of Eq. (9) can be described

$$\frac{\partial F(x,y)}{\partial x} = -\frac{\partial \eta}{\partial x} \tag{10}$$

$$\frac{\partial F(x,y)}{\partial y} = 1 \tag{11}$$

$$\frac{\partial F(x,y)}{\partial z} = 0 \tag{12}$$

$$|\nabla F| = \sqrt{1 + \left(\frac{\partial \eta}{\partial x}\right)^2} \tag{13}$$

Substituting Eqs. (10)–(13) into Eq. (3), the normal vector, $\mathbf{n}$, can be described as

$$\mathbf{n}_1 = \begin{pmatrix} \frac{-\partial\eta/\partial y}{\left[1+(\partial\eta/\partial y)^2\right]^{\frac{1}{2}}} \\ \frac{1}{\left[1+(\partial\eta/\partial y)^2\right]^{\frac{1}{2}}} \\ 0 \end{pmatrix} \tag{14}$$

The curvature of the interface is defined as

$$\kappa = \nabla \cdot \mathbf{n}_1 = \frac{\partial n_{1x}}{\partial x} + \frac{\partial n_{1y}}{\partial y} + \frac{\partial n_{1z}}{\partial z} \tag{15}$$

The term of $\frac{\partial n_{1x}}{\partial x}$ in Eq. (15) can be described as

$$\frac{\partial n_{1x}}{\partial x} = \frac{\partial\left(\frac{-\partial\eta/\partial x}{\left[1+(\partial\eta/\partial x)^2\right]^{\frac{1}{2}}}\right)}{\partial x} = \frac{\left[1+(\partial\eta/\partial x)^2\right]^{\frac{1}{2}} \cdot \left(-\frac{\partial^2\eta}{\partial x^2}\right) + \frac{1}{2}\cdot(\partial\eta/\partial x)\cdot\frac{2\cdot(\partial\eta/\partial x)\cdot\frac{\partial^2\eta}{\partial x^2}}{\left[1+(\partial\eta/\partial x)^2\right]^{\frac{1}{2}}}}{1+(\partial\eta/\partial x)^2} \tag{16}$$

Eq. (16) can be rewritten as

$$\frac{\partial n_{1x}}{\partial x} = \frac{-\frac{\partial^2\eta}{\partial x^2}}{\left[1+(\partial\eta/\partial x)^2\right]^{\frac{3}{2}}} \tag{17}$$

Other terms in Eq. (15) can be described as

$$\frac{\partial n_{1y}}{\partial y} = 0 \tag{18}$$