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The effect of ambient temperature on the stability of thermocapillary flow in liquid column

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ABSTRACT

The hydrodynamic stability of a thermocapillary flow in a finite-size liquid bridge, with a non-deformable interface, heated from above and surrounded by a passive gas is examined by means of three-dimensional direct numerical simulations. Convective flow in 1 cSt silicone oil with $Pr = 18$ is driven by combined effects of buoyancy and thermocapillary forces. Effects of the interfacial heat exchange on hydrothermal stability of the bulk flow are evaluated for various external thermal conditions in the gas phase. Cooling the interface can significantly shift the bifurcation point where the thermocapillary flow becomes oscillatory. If the Biot number Bi, which is a measure of the rate of the heat exchange, is not large, the effect of the interfacial heat flux can be either stabilizing or destabilizing, depending on both the temperature profile in the gas and the height of the liquid bridge. For a high value of heat loss rate, stabilization occurs regardless of the temperature distribution in the ambient gas. Various flow regimes are identified, and detailed stability maps are made for the flow under the considered ambient thermal conditions. It was found that the critical azimuthal mode of the flow changes according to the surrounding conditions in the gas phase. Observations have revealed three distinct azimuthal modes: $m = 0$ (critical for a long liquid bridge at small Biot numbers), $m = 1$ and $m = 2$. Any mode change of the flow is accompanied by an abrupt change of the frequency of temperature oscillations.

1. Introduction

Marangoni flows are caused by tangential stresses due to inhomogeneity of surface tension. Any gradient of the surface tension due to, e.g., nonuniform temperature along the interface separating two different fluids inevitably generates a large-scale interfacial motion (called *thermocapillary flow*). Such flows are very common in many industrial applications, e.g., in crystal growth, evaporation, welding. The geometry of interest in this study is a non-isothermal cylindrical liquid bridge (*LB*), which is a liquid volume suspended between two differentially heated coaxial and parallel disks. The temperature dependence of surface tension leads to thermocapillary Marangoni stresses acting along the cylindrical free surface. These stresses are balanced by viscous stresses and thus the liquid will be set in motion. The flow at the interface is directed downward from the hot disk to the cold one. This flow is often used to model the process of crystal growth by the floating zone technique. At small temperature difference the flow is a steady axisymmetric toroidal vortex (called *base state*). However, when $\Delta T = T_h - T_c$ exceeds some critical value ΔT_{cr}, instability sets in and gives rise to a number of time-dependent three-dimensional flow regimes. It was first experimentally observed by Schwabe and Scharmann [1] and by Chun and Wuest [2].

In liquid bridges made of high Prandtl number fluids (the Prandtl number $Pr = \nu/k$ is the ratio of the kinematic viscosity ν to the thermal diffusivity k) hydrothermal instability is oscillatory and emerges through a supercritical Hopf bifurcation either as a wave traveling in the azimuthal direction or a standing wave. The three-dimensional supercritical flow is characterized by formation of a spatially organized pattern with integer number of pairs of hot and cold cells (see e.g., [3]). The properties of the wave are governed by the physical parameters of the system, such as strength of buoyancy [4,5], height of the column [6,7] and liquid volume [8,9].

In industrial applications related to crystal growth, the oscillatory thermocapillary convection was often accused of being responsible for periodic concentration variations in single crystals (see e.g., [10]). Aiming to understand the onset of oscillatory instability and suppress it, much efforts were put into studying convection in liquid bridge both theoretically [11–13] and experimentally [6,14]. The critical temperature difference, or suitably defined as the critical *thermocapillary* Reynolds number $Re_{cr} \propto \Delta T_{cr}$, as well

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as the wave number m of the supercritical flow were calculated and measured for different liquids, including liquid bridges with a non-cylindrical free surface [8,9].

Experimental evidence of the important role of heat transport through the liquid–gas interface in the stability of the thermocapillary flow in *LB* has been reported since the 1980's. Dressler and Sivakumaran [15] performed experiments on silicone oil *LB*. They used a vertical jet of air blown tangentially along a free surface for producing a viscous shear drag opposing the Marangoni shear at the free surface. An average reduction of Marangoni velocities by 66% was reported in course of their experiments.

Velten et al. [16] carried out experiments in liquid bridges with $Pr = 1, 9$ and 49. They somewhat confirmed the observations of [15] that air motion around the liquid column has a strong effect on the onset of an oscillatory flow. This work reported that values of ΔT_{cr} were higher while heated from below. The difference was attributed to alterations of the flows in the gas surrounding *LB*, which was confined by a large quartz cylinder. When heated from above, this gas flow exhibits a pair of counter-rotating toroidal convective cells, which modify the radial heat transfer.

The increase of the critical temperature difference in case of heating the liquid bridge from below was later confirmed by Lappa et al. [7]. They also reported on a smaller wave number m and found that standing wave regime becomes more stable.

Kamotani et al. [17,18] experimentally measured effects of surface heat loss/gain for 2 and 5 cSt silicone oils with high Prandtl numbers ($Pr = 24 - 49$). They performed experiments in *LB* using 2 and 3 mm diameter rods and thus reaching aspect ratios $\Gamma = d/R = 0.8 - 1.4$. A wide range of ambient temperatures were studied by placing the experimental setup into an oven. These authors estimated the Biot number to be equal to about unity or smaller. Even at such moderate values of Bi the enhanced heat loss from the surface significantly destabilized the Marangoni flow. The critical temperature difference changed by a factor of two to three by changing the air temperature.

Shevtsova et al. [19] and Mialdun and Shevtsova [20] experimentally investigated different thermal conditions around the interface of a liquid bridge of aspect ratio $\Gamma = 1.2$ made of 5 and 10 cSt silicone oils. The experiments were carried out for a wide range of liquid volumes. They varied temperature profile in air performing experiments with and without external shielding. They reported on a remarkable influence of temperature distribution in air on the stability of a thermocapillary flow.

Recent experimental benchmark data of [21] has shown that a poor control of the conditions in the ambient gas phase leads to relatively large uncertainties in the determination of the critical Marangoni number for the onset of a three-dimensional flow.

Over the past decades, multiple numerical studies of thermal convection in liquid bridges with interfacial heat transfer have been performed. Xu and Davis [22] considered an infinitely long liquid bridge under weightlessness. Maintaining a constant temperature of gas and choosing either $Bi = 0$ or 1 they predicted an increase of the critical Marangoni number with the increase of the Biot number. Recently, a linear stability analysis of a thermocapillary flow in an infinitely long cylinder was performed with a co-axial gas flow [23]. It was shown that the gas flow co-directed with the interface thermocapillary motion has a destabilizing effect on the system. Gas moving in the opposite direction can be stabilizing or destabilizing depending on the gas flow rate.

Three-dimensional numerical simulations were conducted in *LB* surrounded by gas of constant temperature [24] for a large aspect ratio, $\Gamma = 1.8$ and $Pr = 18$. The authors reported that increasing the Biot number from zero up to 1.8 does decrease the critical Marangoni number by 33%. For the same aspect ratio the effect of interfacial heat exchange on the stability of the thermocapillary flow in *LB* with $Pr \approx 16$ was investigated by the linear stability analysis under normal gravity [25] and weightlessness [26]. For both constant and linear temperature distribution in gas, the computed results provided a stability curve with convex trend as a function of the Biot number.

Summarizing the above discussion we may underline, that the impact of heat loss (gain) on the stability of a thermocapillary flow is strongly determined not only by the Biot number and the ambient temperature in gas, but is also influenced by the Prandtl number, the aspect ratio, the gravity level, and the flow in the gaseous phase [19,27].

In the present work, we analyze the influence of heat exchange between liquid and air on both the stability limits and the spatial organization of the thermocapillary flow. Our analysis highlights two essential points. First, the effect of temperature distribution in the surrounding gas on the hydrodynamic stability of a thermocapillary flow in a liquid bridge. Second, the impact of the Biot number on the stability of convective flows for a large range of aspect ratios. In some way this work is a continuation of the study reported in [28].

This paper is organized as follows: in Section 2 we formulate the model for the problem of coupled buoyancy and Marangoni convection, and explain how the heat transfer through liquid–gas interface is modeled. Results of the numerical simulations and their discussion are presented in Section 3. We first consider a liquid column with aspect ratio 1.8 and, by varying both the heat transfer rate and the ambient thermal conditions in gas, show how the heat flux through the interface influences gross, i.e., integral, features of the flow and how it affects the critical conditions (the critical Reynolds number, mode of the flow and the frequency of temperature oscillations). Finally, we show how ambiguous the effect of heat transport on the onset of thermocapillary instability is. This result follows from the comparison between the instability diagrams obtained for liquid bridges of different aspect ratios.

2. Problem formulation

2.1. Governing equations and parameters

A liquid drop is suspended between two parallel, coaxial rigid disks of radius R separated by a distance d and kept at a temperature difference $\Delta T = T_h - T_c$ ($T_h > T_c$). The liquid is a Newtonian fluid with temperature-dependent properties. The density ρ, surface tension σ, and kinematic viscosity ν of the liquid are linearly decreasing functions of temperature

$$\rho(T) = \rho_0 - \rho_0\beta(T - T_0), \quad \beta = -\frac{1}{\rho_0}\frac{\partial\rho}{\partial T},$$
$$\sigma(T) = \sigma_0 - \sigma_T(T - T_0), \quad \sigma_T = -\frac{\partial\sigma}{\partial T},$$
$$\nu(T) = \nu_0 + \nu_T(T - T_0), \quad \nu_T = \frac{\partial\nu}{\partial T}.$$

here the subscript index 0 indicates the quantities at the reference temperature $T_0 = T_c$.

The liquid–gas interface is assumed to be cylindrical and nondeformable, as shown in Fig. 1. We consider the limit of asymptotically large mean surface tension σ_0 and a liquid volume of $\pi R^2 d$. In this limit, the liquid bridge takes an upright cylindrical shape which is not influenced by dynamic pressure.

The dimensionless equations governing this system are the incompressible Navier–Stokes equations in the Boussinesq approximation, the continuity and the thermal energy equations:

$$\frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} = -\nabla P + R_\nu\cdot 2\mathbf{S}\times\nabla\Theta + (1 + R_\nu\Theta)\nabla^2\mathbf{V} + \vec{e}_z Gr\,\Theta, \quad (1)$$

$$\nabla\cdot\mathbf{V} = 0, \quad (2)$$

$$\frac{\partial\Theta}{\partial t} + \mathbf{V}\cdot\nabla\Theta = \frac{1}{Pr}\nabla^2\Theta, \quad (3)$$