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Two immiscible layers of electro-osmotic driven flow with a layer of conducting non-Newtonian fluid

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ABSTRACT

Most of the bio-fluids are non-Newtonian fluid with complex flow behavior, for instances the human blood and DNA sample are shear thinning fluids. The transportation of such fluids shares wide applications in microfluidics. The mathematical model for the two immiscible layers electro-osmotic driven flow in the parallel microchannel is proposed in this paper. One layer is conducting non-Newtonian power-law fluid driven by electro-osmotic force. The other layer is non-conducting Newtonian layer driven by interface shear. The effects of Debye–Huckel parameter κh_1, interfacial zeta potential ψ_I, the Newtonian viscosity μ_2, the non-Newtonian fluid consistency coefficient m & flow behavior index n are discussed. The complex flow behavior, namely fluid consistent coefficient and flow behavior index, play important roles upon the velocity distributions. The shear thinning effect is also analyzed. The results show that the shear thinning fluid is not only ideal for direct electro-osmotic driving but also for hybrid driving.

1. Introduction

Electro-osmotic driven flow gets wide applications in microfluidics, among which the electro-osmotic (EO) pump utilizes the electro-osmotic force as the driven force. The drawback is that the non-conducting fluid cannot be driven by the electro-osmotic force directly. The idea of driving the non-conducting fluid by viscous shear stress of the conducting fluid is proposed (hybrid EO pump) [1]. Gao et al. [2,3] and Li et al. [4,5] conducted study on two or multi layers immiscible fluids driven by electro-osmotic effect and the mathematical approach is attained by solving the Poisson–Boltzmann and Navier–Stokes equations.

The non-Newtonian fluid based applications, such as DNA sample transportation, separation, mixing of “Lab on chip”, are in high demand and received intense focus [6–8]. Due to the complexity of non-Newtonian behavior, theoretical investigation on the non-Newtonian electro-osmotic flow is limited. Currently, theoretical research mainly concentrates on polymer solution, with which the bio-sample behaviors similarly [9–12]. Das et al. [13] first derived the theoretical model for a flat plane (or parallel plane). By utilizing the Poisson–Boltzmann equation, momentum and energy equations, the velocity profile and temperature distribution are obtained for the power-law non-Newtonian fluid. Zhao et al. [14,15] derived an exact solution for a single layer EOF driven power-law non-Newtonian fluid with symmetrical electric double layer (EDL) condition in microchannels. The explicit form of velocity profile is obtained. Key parameters such as the flow behavior index n, the Debye–Huckel parameter κ are studied. Zhao et al. further expanded the mathematical model by including the Gouy–Chapman solution to the Poisson–Boltzmann equation and the Carreau fluid constitutive model [16]. The non-linear ordinary equations were solved numerically and a more general model was obtained. The Carreau fluid model performs well and the Weissenberg number (Wi) is also studied. Vasu et al. [17] expanded the single layer EOF power-law model by solving the EDL potential distribution at high zeta potential, upon which the Debye Huckel linear approximation is no longer valid. The numerical results are obtained.

The work stated above mainly focus on the direct electro-osmotic effect upon one single layer conducting non-Newtonian fluid. The fluids described by the model are types of non-Newtonian fluids with effective electro-osmotic and shear thinning effect. Non-ionic polymer solutions are ideal samples as the electro-osmotic effect can be controlled by adding salts and polymer solutions show obvious shear thinning effect. This paper aims to provide the theoretical analysis of two immiscible layers of electro-osmotic driven flow with one layer of conducting non-Newtonian fluid. The two layer model is capable of driving the non-conducting fluid via interface shear. The constitutive equation of non-Newtonian layer fluid is described by power-law. The shear thinning effect which prompts

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the flow rate by reducing the shear rate dependent viscosity makes the shear thinning fluid ideal for hybrid EO pump.

2. Mathematical model

Fig. 1 shows the model of the two immiscible layers flow between parallel plates. There are two physical layers, non-Newtonian layer 1 and Newtonian layer 2 represented by h_1 and h_2, respectively (Fig. 1(a)).

The upper layer is a conducting non-Newtonian layer described by power-law, driven by an applied external electric field. The lower layer is the non-conducting Newtonian layer driven by the interface viscous shear stress. To develop the analytical model, the non-Newtonian layer h_1 is sub-divided into different regions as indicated in Fig. 1(b).

The heights of the non-Newtonian and Newtonian layers are h_1 and h_2, respectively. The zeta potentials on the wall and the interface are ψ_W and ψ_I, respectively.

2.1. Electric potential distribution in the conducting layer

The electric potential distribution only exists within the conducting non-Newtonian fluid and it follows the Poisson–Boltzmann equation:

$$\nabla^2\psi = -\frac{\rho_e}{\varepsilon} \tag{1}$$

where, ψ is the electric potential in the solution, ρ_e is the electric charge density, ε is the electric permittivity of the solution.

The electric charge density can be expressed as:

$$\rho_e = -2z_v e_0 n_0 \sinh\left(\frac{z_v e_0}{\kappa_b T}\right) \tag{2}$$

where, n_0 is ionic concentration in the bulk solution, e_0 is the fundamental electric charge, z_v is the valence of the ion, ε is the electric permittivity of the solution, κ_b is the Boltzmann constant, T is the absolute temperature.

Dealing with the parallel plate model, the equation can be simplified as:

$$\frac{d^2\psi}{dy^2} = \frac{2z_v e_0 n_0}{\varepsilon}\sinh\left(\frac{z_v e_0}{\kappa_b T}\right) \tag{3}$$

Equation (3) can be linearized by using the Debye–Huckel approximation, $\sinh\left(\frac{z_v e_0}{k_b T}\right) = \frac{z_v e_0}{k_b T}$. It physically means: $|z_v e_0\psi| < \kappa_b T$. We can obtain the linearized equation for the electric potential distribution:

$$\frac{d^2\psi}{dy^2} = \kappa^2\psi \tag{4}$$

$$\kappa^2 = \frac{2z_v^2 e_0^2 n_0}{\varepsilon k_b T} \tag{5}$$

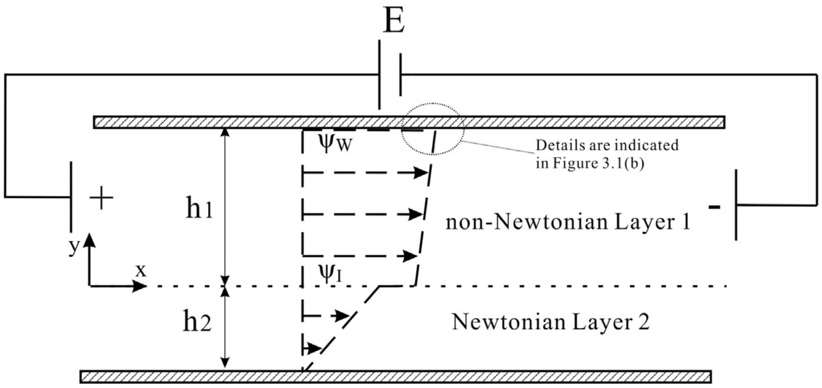
where κ is the Debye–Huckel parameter and κ^{-1} is normally considered as the thickness of electric Debye layer (EDL).

Boundary conditions for the electric potential:

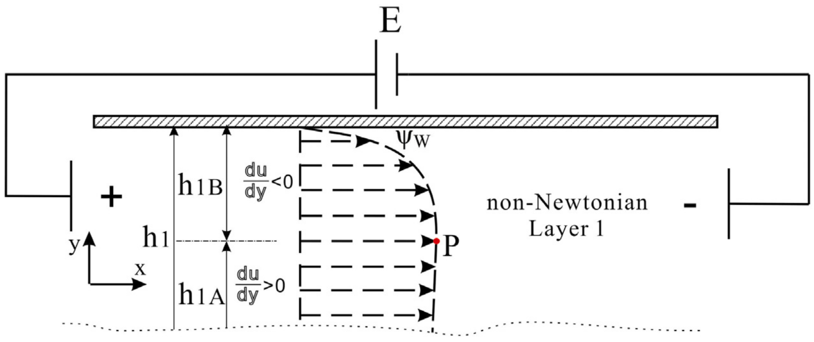
Wall boundary zeta potential at $y = h_1 : \psi = \psi_W$
Interface boundary zeta potential at $y = 0 : \psi = \psi_I$

The solution for the electric potential (Eq. (4)) under the boundary conditions is:

$$\psi = \frac{\sinh(\kappa y)}{\sinh(\kappa h_1)}\psi_W + \frac{\sinh(\kappa h_1 - \kappa y)}{\sinh(\kappa h_1)}\psi_I \tag{6}$$



(a) Two-fluid model over the whole channel



(b) Details in the non-Newtonian layer 1 near the wall boundary

Fig. 1. Schematic diagram of two immiscible layers electro-osmotic driven flow model.