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An effective approach for transient thermal analysis in a functionally graded hollow cylinder

H.M. Wang *

Department of Engineering Mechanics, School of Aeronautics and Astronautics, Zhejiang University, Hangzhou 310027, PR China

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ABSTRACT

An effective approach is developed to analyze the transient thermal analysis in a functionally graded hollow cylinder. The heat conductivity, mass density and specific heat are assumed to vary along the radial direction with arbitrary grading pattern. The transient solution in the radial direction is obtained based on the laminate approximation theory. In solving technique, the transient solution is divided into two parts: One is quasi-static solution and the other is dynamic solution. The quasi-static solution is obtained by the state space method and the dynamic solution is obtained by the initial parameter method. In this analysis, all the manipulation is performed in time domain. In the illustrative examples, the effect of the inhomogeneity form on the transient thermal responses is investigated.

1. Introduction

Thermal analysis is an important topic in thermal engineering. As a result of temperature different, the heat conduction and energy transfer may take place in or between the material bodies. Thermal process extensively exists in manufacturing process, automobile engineering, aerospace engineering, chemical pipe, nuclear engineering and living tissues [1,2].

Functionally graded materials (FGMs) are a new class of materials with the material properties vary continuously along the specified directions. Due to their desirable properties, FGMs have been increasing used in modern engineering applications. For example, the functionally graded metal-ceramic composites have been used as thermal barriers or thermal shields in various applications [3,4]. Especially, in severe temperature environments, such as extremely high temperature and thermal shock, widely potential applications are opening for FGMs [5,6].

The theoretical analysis is always important in understanding the thermal behaviors of new materials and structures when they serve in thermal environments. A number of research works have been carried out for heat conduction and thermal analysis in cylindrical structures. For steady state problem, Bahadur and Bar-Cohen [7] obtained a closed form analytical solution of a cylindrical pin fin with orthotropic thermal conductivity. Mustafa et al. [8] developed a closed-form analytical solution of orthotropic annular fins with contact resistance. Kayhani et al. [9] presented an exact solution of conductive heat transfer in cylindrical composite laminate in the radial and azimuthal directions. Kayhani et al. [10] further obtained a general analytical solution for heat conduction in cylindrical multilayer composite laminates in the radial and axial directions. Tarn and Wang [11] studied the end effects of heat conduction in circular cylinders of functionally graded materials and laminated composites. Hosseini and Abolbashari [12] presented a unified formulation to analyze of temperature field in a thick hollow cylinder made of functionally graded materials with various grading patterns.

A number of analytical investigations have also been carried out for transient problem. Lu et al. [13] obtained the transient analytical solution to heat conduction in finite composite circular cylinder. The transient heat conduction in polar coordinates with multiple layers in radial direction has been solved by Singh et al. [14]. Asgari and Akhlaghi [15] investigated the transient heat conduction in two-dimensional functionally graded hollow cylinder with finite length. Delouei et al. [16] dealt with the unsteady axisymmetric conductive heat transfer in cylindrical orthotropic composite laminates.

It is noted that exact solution can be obtained only for some special grading patterns [11,17–19]. For transient thermal analysis, usually Laplace transform is employed in the solving technique [13,16]. Some presented methods are valid only for constant boundary conditions [14,19]. For most cases, it is almost impossible to obtain the exact solutions of heat conduction in functionally graded cylindrical structures. To discover the basic thermophysical process or to gain advantage properties of FGMs, it is necessary to carried out the precise evaluation of thermal analysis in functionally graded materials.

* Tel.: +86 571 8795 2396; fax: +86 571 8795 2570.
E-mail address: wanghuiming@zju.edu.cn

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Nomenclature

a, b	inner and outer radii, m
r	radial coordinate, m
t	time, s
T	temperature difference, K
T_0	reference temperature, K
T_a, T_b	ambient temperature inside and outside the cylinder, K
k_a, k_b	thermal conductivity at the internal and external surfaces, W/m K
ρ_a, ρ_b	mass density at the internal and external surfaces, kg/m^3
c_a, c_b	specific heat at the internal and external surfaces, J/kg K
h_a, h_b	heat transfer coefficient at the internal and external surfaces, W/m^2 K
$k(r)$	thermal conductivity, W/m K
$\rho(r)$	mass density, kg/m^3
$c(r)$	specific heat, J/kg K
q_r	heat flux, W/m^2
N	number of the fictitious layers
r_i	radius of the ith interface, m
T_i	temperature of the ith layer, K
k_i	thermal conductivity of the ith layer, W/m K
ρ_i	mass density of the ith layer, kg/m^3
c_i	specific heat of the ith layer, J/kg K
$q_{r,i}$	heat flux at the ith interface, W/m^2
ξ	nondimensional radial coordinate
ξ_i	nondimensional radius of the ith interface
η_i	nondimensional radius of the middle surface of the ith layer
$R_{i,m}(\xi)$	eigenfunction
J_0, Y_0	Bessel functions of the first and the second kinds of order 0
J_1, Y_1	Bessel functions of the first the second kinds of order 1
λ_m	mth eigenvalue
$\bar{T}_i$	nondimensional temperature of the ith layer
$\bar{T}_{s,i}$, $\bar{T}_{d,i}$	nondimensional temperature of the quasi-static and dynamic solutions of the ith layer
$\bar{q}_{r,i}$	nondimensional heat flux of the ith layer
$\bar{q}_{s,i}$, $\bar{q}_{d,i}$	nondimensional heat flux of the quasi-static and dynamic solutions of the ith layer
$\bar{k}_i$, $\bar{\rho}_i$, $\bar{c}_i$	nondimensional thermal conductivity, mass density and specific heat of the ith layer
$\bar{\alpha}_i$	nondimensional thermal diffusivity of the ith layer
$\bar{k}_a$, $\bar{k}_b$	nondimensional thermal conductivity at the internal and external surfaces
$\bar{h}_a$, $\bar{h}_b$	nondimensional heat transfer coefficient at the internal and external surfaces
Exp	exponential function
τ	nondimensional time
$\mathbf{I}$	identity matrix
μ_1, μ_2	material inhomogeneity parameters

Subscripts

i	ith layer
m	mth eigenvalue and eigenfunction
s	index for quasi-static solution
d	index for dynamic solution

In this investigation, a new approach is developed for transient thermal analysis in functionally graded hollow cylinder based on the laminate approximation theory. Along the radial direction, the grading pattern of the material properties can be arbitrary. The presented method is valid for both constant and time-dependent boundary conditions.

2. Mechanical model and basic equations

Consider a functionally graded hollow cylinder of inner and outer radii a and b, respectively, as shown in Fig. 1. The polar coordinate system is employed in the following analysis. The thermal conductivity k, mass density ρ and the specific heat c are assumed to be functions of radial position r:

$$k = k(r), \quad \rho = \rho(r), \quad c = c(r). \tag{1}$$

For one dimensional transient heat conduction, the governing equations are [20]

$$q_r = -k(r)\frac{\partial T(r,t)}{\partial r}, \quad -\frac{1}{r}\frac{\partial}{\partial r}(rq_r) = \rho(r)c(r)\frac{\partial T(r,t)}{\partial t}. \tag{2}$$

The boundary conditions are prescribed as

$$\begin{aligned} r = a: &\quad -k_a\frac{\partial T(r,t)}{\partial r} + h_a[T(r,t) - T_a(t)] = 0, \\ r = b: &\quad k_b\frac{\partial T(r,t)}{\partial r} + h_b[T(r,t) - T_b(t)] = 0. \end{aligned} \tag{3}$$

The current problem is difficult to be solved due to the material inhomogeneity along the radial direction. In this investigation, the laminate approximation theory is employed to overcome this difficulty. The functionally graded (FG) hollow cylinder is fictitiously treated as a laminate composites cylinder made of N sub-layers. The inner and outer radii of the ith layer are denoted as r_{i-1} and r_i, respectively. For each sub-layer ($r_{i-1} \leqslant r \leqslant r_i$, $i = 1,2,\ldots,N$), the material properties are assumed to be constant and are taken as the values at its internal surface $r = r_{i-1}$:

$$k_i = k(r_{i-1}), \quad \rho_i = \rho(r_{i-1}), \quad c_i = c(r_{i-1}). \tag{4}$$

Then the heat conduction equation for each sub-layer can be written as

$$q_{r,i} = -k_i\frac{\partial T_i(r,t)}{\partial r}, \quad -\frac{1}{r}\frac{\partial}{\partial r}(rq_{r,i}) = \rho_i c_i\frac{\partial T_i(r,t)}{\partial t}. \tag{5}$$

Then Eq. (3) becomes

$$\begin{aligned} r = a: &\quad -k_0\frac{\partial T_1(r,t)}{\partial r} + h_a[T_1(r,t) - T_a(t)] = 0, \\ r = b: &\quad k_N\frac{\partial T_N(r,t)}{\partial r} + h_b[T_N(r,t) - T_b(t)] = 0. \end{aligned} \tag{6}$$

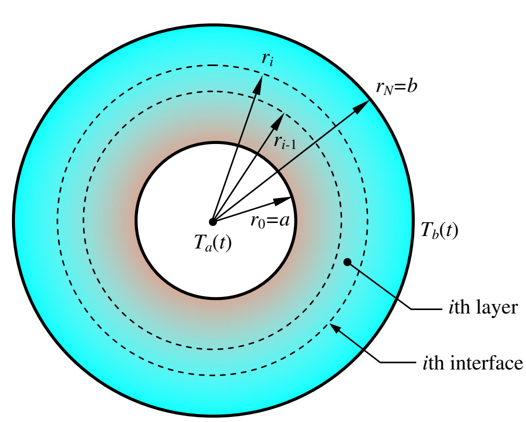

Fig. 1. Mechanical model of functionally graded hollow cylinder.