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Constructal entransy dissipation rate minimization for tree-shaped assembly of fins

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ABSTRACT

Based on the total volume and fin-material volume constraints, a tree-shaped assembly of fins is optimized with entransy dissipation rate minimization, and the dimensionless mean thermal resistance and the corresponding optimal construct are obtained. The results show that the heat conductance performance of the assembly is better as the fraction of fin material increases, thermal conductivity of fin grows smaller, and the heat transfer coefficient of fin over all the exposed surfaces becomes larger. The heat conductance performance of the tree-shaped assembly is not always better with more complexity, and is far better than that of the T-shaped assembly. The optimal constructs corresponding to the minimization of entransy dissipation rate and the minimization of maximum thermal resistance, respectively, are different obviously. For the former, its dimensionless mean thermal resistance is smaller, and the heat conductance performance is much better.

1. Introduction

Recently, heat transfer optimization has been performed by using constructal theory [1–32]. Bejan [1] first put forward the constructal theory and applied it to heat conduction optimization in 1996. Since then, the constructal theory that was applied to heat transfer optimization problems of fins [33–47] has been developing rapidly.

The heat transfer optimizations of fins in Refs. [33–47] reflect the temperature limitation of the fins with the maximizations of heat transfer rates or the minimizations of maximum thermal resistances. To reflect global heat conduction performance and improve the heat transfer efficiency, Guo et al. [48,49] proposed a new physical quantity called "entransy" describing the heat transfer ability and the entransy dissipation extremum principle. The physical meaning of entransy was further expounded according to the researches such as physical mechanisms of heat conduction and electro-thermal simulation experiments. The entransy dissipation has caught much attention of some authors studying all kinds of heat transfer optimization problems [50–65]. By combining the entransy dissipation extremum principle [48–65] with constructal theory [1–47], a series of constructal optimizations have been conducted by Refs. [66–81].

Bejan and Almogbel [35] adopted analytical method and took heat transfer rate as optimization objective to conduct constructal optimizations of T-, τ- and umbrella-shaped assemblies of fins. Almogbel [36] studied the tree-shaped assembly composed of many T-shaped assemblies of fins, and showed that the tree-shaped assembly could further enhance its heat conductance. It is not enough to optimize the fins with the local optimization objectives such as the maximizations of heat transfer rates (or the minimizations of maximum thermal resistances). Therefore, based on Ref. [35], the minimizations of entransy dissipation rates for T- and umbrella- shaped assemblies of fins were conducted in Refs. [76,81]. This paper will study a tree-shaped assembly of fins with entransy dissipation rate minimization based on Ref. [36], and the obtained results will be compared with theose of T-shaped assembly.

2. Tree-shaped assembly

Consider a tree-shaped second-order assembly composed of a number (n) of T-shaped assemblies shown as in Fig. 1 [36]. Such T-shaped first-order assembly is assembled with two elemental fins ($L_{bi} \times t_{bi}$) and one first-order fin ($L_i \times t_i$). Just as in Ref. [36], this paper sets that $L_{b1} = \cdots = L_{bi} = \cdots = L_{bn}$, $t_{b1} = \cdots = t_{bi} = \cdots = t_{bn}$, $L_1 = \cdots = L_i = \cdots = L_n$ and $t_1 = \cdots = t_i = \cdots = t_n$, where b$i$, n, i, k, h, q_B,

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Nomenclature

A	area of the fin allocated to the space
B	Biot number
h	convective heat transfer coefficient
L	length of fin
n	number of branch pairs
R	mean thermal resistance
T	temperature
t	width of fin
θ	$\theta_i = (T - T_\infty)/(T_{q_{1/2}} - T_\infty)$
a	$a = (2h\ A^{1/2}/k)^{1/2}$
E	entransy dissipation rate
k	thermal conductivity
m	$m = \left(\frac{2h}{kt}\right)^{1/2}$
q	heat transfer rate
S	slenderness, $S = \sum_{i=1}^{n} L_i/(2L_{bn})$
T_∞	fin surface temperature
ϕ	fraction of the fin material

Superscript

$\sim$	nondimensionalization
$\cdot$	rate

Subscript

B	branch
i	ith branch pair
f	fin surface
h	entransy dissipation rate

and T_∞ all have the same meanings as in Ref. [36]. The differences from Ref. [36] are that q_B is specified and T_B is unknown.

In this paper, constructal optimization of tree-shaped assembly is to determine the optimal geometry (the number (n) of branch pairs, the slenderness of the tree-shaped assembly $(\sum_{i=1}^{n} L_i/(2L_{bn})$ and the branch-to-stem aspect ratio (t_{bi}/t_i)) with the minimization of entransy dissipation rate. As in Ref. [36], the constructal optimization of this paper is subjected to two constraints, namely, the total volume constraint,

$$A = 2L_n \sum_{i=1}^{n} L_i = \text{constant} \tag{1}$$

and the fin-material volume constraint,

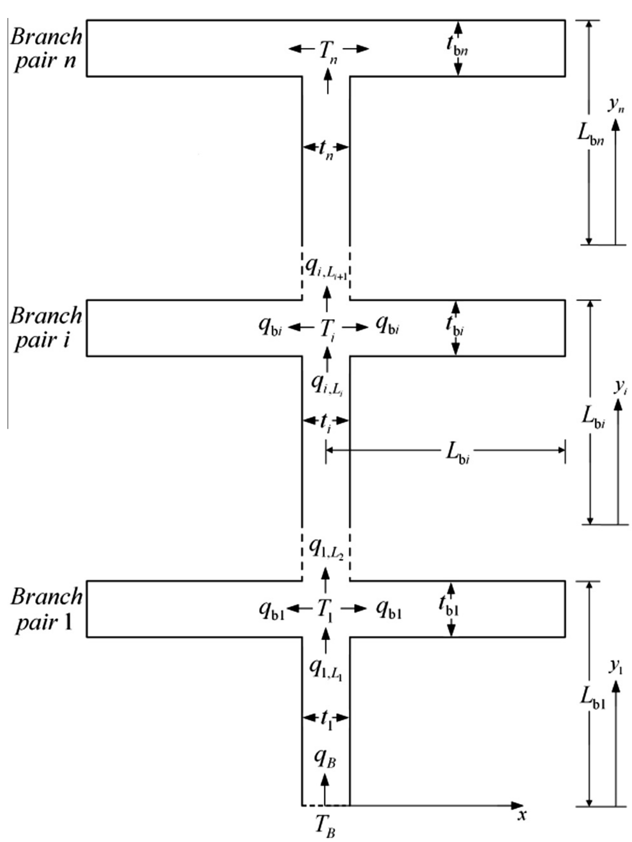

Fig. 1. Tree-shaped assembly of fins [36].

$$A_f = \sum_{i=1}^{n} (L_i t_i + 2L_{bi} t_{bi}) = \text{constant} \tag{2}$$

where the subscript f denotes the fin-material. Eq. (2) can be converted to the fin-material fraction which is the fraction of fin material allocated to the space, $\phi_2 = A_f/A \ll 1$ is defined, where the subscript 2 denotes a tree-shaped second-order assembly. Assume that heat transfer thermal resistance of the fin surface, $1/h$, is much larger than heat conductance thermal resistance, t_i/k and t_{bi}/k $(i = 1, 2, \ldots, n)$. That is, the following Biot number criterion is satisfied:

$$\begin{aligned} B_i &= (ht_i/k)^{1/2} \ll 1 \\ B_{bi} &= (ht_{bi}/k)^{1/2} \ll 1 \end{aligned} \tag{3}$$

So the temperature of the fin at each cross-section can be regarded as uniform. Now the heat conduction directions of elemental fins and the rib may be considered as one-dimensional directions along the L_{bi} and L_i fins, respectively.

For the ith T-shaped assembly the temperature distribution in one elemental fin is [36]

$$T_x - T_\infty = (T_i - T_\infty)[\cosh(m_{bi}x) - \sinh(m_{bi}x)\tanh(m_{bi}L_{bi})] \tag{4}$$

where $m_{bi} = \left(\frac{2h}{kt_{bi}}\right)^{1/2}$.

According to entransy dissipation rate defined in Ref. [49], the entransy dissipation rate of one elemental fin is

$$\begin{aligned} \dot{E}_{vh\varphi,bi} &= \int_0^{L_{bi}} Wt_{bi}k\left(\frac{dT_x}{dx}\right)^2 dx \\ &= \frac{akW}{4}(T_i - T_\infty)^2 \mathrm{Sech}(a\tilde{L}_{bi}\tilde{t}_{bi}^{-1/2})^2 \left[-2a\tilde{L}_{bi} + \tilde{t}_{bi}^{1/2}\sinh(2a\tilde{L}_{bi}\tilde{t}_{bi}^{-1/2})\right] \end{aligned} \tag{5}$$

where $(\tilde{L}_{bi}, \tilde{t}_{bi}) = (L_{bi}, t_{bi})/A^{1/2}$ and $a = (2h\ A^{1/2}/k)^{1/2}$.

The temperature distribution in the corresponding stem is [33]

$$\begin{aligned} T_{y_i} - T_\infty = {} & \mathrm{csch}(m_i L_i)[(T_i - T_\infty)\sinh(m_i y_i) - (T_{i-1} - T_\infty) \\ & \times \sinh(m_i y_i - m_i L_i)] \end{aligned} \tag{6}$$

where $m_i = \left(\frac{2h}{kt_i}\right)^{1/2}$, y_i denotes the y coordinate established in the ith T-shaped assembly. Because $m_i L_i = a\tilde{L}_i\tilde{t}_i^{-1/2}$, the entransy dissipation rate of the corresponding stem is

$$\begin{aligned} \dot{E}_{vh\varphi,i} = {} & \int_0^{L_i} Wt_i k\left(\frac{dT_{y_i}}{dy}\right)^2 dy = \frac{akW}{4}\mathrm{csch}(a\tilde{L}_i\tilde{t}_i^{-1/2})^2 \left\{2a\tilde{L}_i[(T_i - T_\infty)^2 \right. \\ & - 2\cosh(a\tilde{L}_i\tilde{t}_i^{-1/2})(T_i - T_\infty)(T_{i-1} - T_\infty) + (T_{i-1} - T_\infty)^2] \\ & + 2\tilde{t}_i^{1/2}\sinh(a\tilde{L}_i\tilde{t}_i^{-1/2})\left[\cosh(a\tilde{L}_i\tilde{t}_i^{-1/2})(T_i - T_\infty)^2 - 2(T_i - T_\infty)\right. \\ & \left.\left. \times \left(T_{i-1} - T_\infty\right) + \cosh(a\tilde{L}_i\tilde{t}_i^{-1/2})(T_{i-1} - T_\infty)^2\right]\right\} \end{aligned} \tag{7}$$