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Energy growth of disturbances in a non-Fourier fluid

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ABSTRACT

Natural convection of non-Fourier fluid of the single-phase-lagging (SPL) type between two horizontal walls (Rayleigh–Benard) has been investigated. These fluids possess a relaxation time, reflecting the delay in the response of the heat flux and the temperature gradient with respect to one another. By invoking the role of the eigenvectors to detect and quantify short-time behavior, transient growth of energy of disturbances has been illustrated. The energy of the perturbations is introduced in terms of the primary variables as a disturbance measure in order to quantify the size of the disturbance. In contrast to linear stability analysis, one does not assume exponential time dependence, but monitor the evolution of initial conditions in the pre- and post-critical ranges of Rayleigh numbers. Different growth functions for different levels of non-Fourier effects have been found, which should be thought of as the envelope of the energy evolution of individual initial conditions. Also, it is found that nonlinearities are not required for the energy growth. Energy growth can occur if non-orthogonal eigenfunctions are available. A fundamental implication of the non-normality is that there can be significant energy growth in the energy of perturbations even if the flow is stable.

1. Introduction

Generally, conduction is considered to be a heat transfer process in which heat is transported in a diffusive way, and is described by Fourier law. When combined with the conservation of energy, Fourier law results in a parabolic equation for the temperature field. It means that if a sample is subjected to a thermal disturbance, the disturbance is felt instantaneously at all points of the sample. Fundamentally, Fourier law is not realistic since a disturbance wave in the temperature will travel at a finite speed as it is transferred by molecular interaction [1]. This behavior is characterized by the Maxwell–Cattaneo or Cattaneo–Vernotte (C–V) equation proposed by Cattaneo [2] and Vernotte [3], which includes a transient term accounting for the finite thermal relaxation time of the medium. This is the time required for the heat flux to relax to a new (stable) steady state following a perturbation in the temperature gradient, establishing a hyperbolic heat (second sound wave) response. It is recalled that what is meant by the *first sound* wave is a propagated disturbance of pressure (or density) through a continuous medium such as air or water. Hearing is the sense of first sound wave perception. The *second sound* is a heat transfer mechanism describing the propagation of heat as a temperature wave.

In practice, most heat transfer problems involve materials with relaxation times on the order of pico- or 10–12 s [4]. In this case, the C–V equation collapses onto the classical Fourier model. However, the recent interest in second sound is due to its potential application in some situations such as the heat transfer in drying sand [5], cooling or heating in stars [6], and in skin burns [7], where the Fourier law is not adequate to describe the heat transfer process. Second sound could also be used in modeling of heat transport in a nuclear fuel rod in a light water reactor [8], and in phase changes [9]. Different models exist to describe non-Fourier heat conduction. Experimental evidence of the wave nature of heat propagation in processed meat was demonstrated by Mitra et al. [10]. The value of the thermal relaxation time of processed bologna meat was found to be on the order of 15 s, which obviously requires the use of the hyperbolic heat conduction model for such a biological material. Antaki [11] used a dual-phase-lag (DPL) model for non-Fourier heat conduction to offer a new interpretation for the experimental evidence in the experiments of Mitra et al. [10] with processed meat. In the DPL model, materials possess a relaxation time and a retardation time, reflecting the delay in the response of the heat flux and the temperature gradient with respect to one another. Unlike the DPL model, in the single-phase-lag model (SPL), materials possess only a relaxation time (the retardation time is zero in this case). Xu et al. [12] developed a computational approach to examine the non-Fourier heat transfer process in skin tissue. They employed the DPL model to study bioheat transfer. Non-Fourier effect has been examined in other applications such as welding [13], laser industry [14,15], biothermomechanics of skin [16], and bio-heat transfer during magnetic hyperthermia treatment [17].

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Nomenclature

∇	gradient operator
Δ	Laplacian operator
$\boldsymbol{V}$	velocity vector
P	pressure
ρ	density
g	gravity acceleration
$\boldsymbol{e}_z$	unit vector in the z direction
μ	viscosity
c_p	specific heat
T	temperature
$\mathbf{Q}$	heat flux vector
τ	relaxation time
p	dimensionless pressure deviations from the base state
θ	dimensionless temperature deviations from the base state
$\boldsymbol{v}(u, w)$	dimensionless velocity vector
$\boldsymbol{q}$	dimensionless heat flux vector
D	length scale (distance between plates)
κ	thermal diffusivity
k	wavenumber in x direction
C	Cattaneo number
j	$\nabla \cdot \mathbf{q}$
Pr	Prandtl number
Ra	Rayleigh number
s	time evolution
n	modenumber
ω	frequency
K	kinetic energy of the perturbations
v_i	perturbation velocity
V	cell volume
dK_R/dt	weighted kinetic energy rate
E	energy norm
$\langle\rangle$	integration over $z \in [0,1]$
G	maximum possible energy growth
$\{s_j\}$	eigenvalues
$\{\boldsymbol{E}_j\}$	eigenvectors
sup	supremum

Subscripts and abbreviations

B	base fluid
i	intersection
m	minimum
H	threshold
c	critical
F	Fourier
z	partial differentiation wrt z
t	partial differentiation wrt t
NP	nanoparticle
NF	nanofluid

Using analytical [18,19] and numerical approaches [17,20–30], non-Fourier effects have been investigated in different geometries such as irregular geometries [21], fins [31], crack tip [32], slabs [33,34], spherical [35] and cylindrical geometry [36]. Also, different boundary conditions are studied such as on-off heat flux boundary condition [37], periodic surface thermal disturbance [19], and axi-symmetric surface sources [38].

Regarding non-Fourier heat conduction in fluids, the first experimental evidence was the detection of the second sound wave (thermal wave) in superfluid helium (He II) at low temperatures ($T < 2.2$ K) [39]. Superfluid helium exhibits outstanding properties such as an extremely high thermal conductivity and the ability to flow through extremely narrow channels without noticeable pressure loss. Shimazaki et al. [40] investigated experimentally transient heat transport phenomena in He II. Their results show that at the lower temperatures, $T < 1.9$ K, the transported energy in the second sound mode becomes more efficient (low temperature phenomenon). Zhang [41] studied numerically the transient thermal wave heat transfer in He II, using the classical C–V thermal wave equation.

Using Maxwell–Cattaneo (or Maxwell–Cattaneo–Fox) heat law, Straughan et al. [42] investigated convective stability in the Benard problem. The problem was modeled using the Jaumann derivative of Fox [43] in the constitutive heat equation, and adopting the Boussinesq approximation in the buoyancy term in the momentum equation. Puri et al. [44] studied non-Fourier heat conduction using the MCF model for the Stokes' first and second problems. The finite thermal relaxation time was found to affect both the temperature and the velocity fields, but this influence is not always consistent. It tends to increase the amplitude of both of these fields under some cases and to decrease it under other cases. Later, Puri et al. [45] used the Maxwell–Cattaneo–Fox (MCF) model to analyze the non-Fourier heat conduction effects in Stokes' first problem for a dipolar fluid. They found that increasing the relaxation time reduces the velocity in heating and increases the velocity in cooling. The temperature predicted by the MCF model is greater than that predicted by Fourier heat law. Note that dipolar fluids are special cases of non-Newtonian fluids with deformable microstructure, consisting of such entities as bubbles, atoms, particulate matter, ions or other suspended bodies.

More recently, Ibrahem et al. [46] studied the nonclassical heat conduction effects in Stokes' second problem of a micropolar fluid, by examining the influences of the thermal relaxation time on angular velocity, velocity field, and temperature. Micropolar fluids possess a microstructure that renders the stress tensor non-symmetric. These fluids consist of randomly oriented particles suspended in a viscous medium such as dust, dirt, ice or raindrops, or other additives. Pranesh et al. [47] studied the Rayleigh–Bénard magneto convection in a micropolar fluid. Using Cattaneo law, Pranesh analyzed the onset of convection. The classical Fourier flux law overpredicts the critical Rayleigh number compared to that predicted by the non-classical hyperbolic law.

In addition, non-Fourier effect can be important in nanofluids [48]. These fluids are solutions consisting of a base fluid solvent, containing a small volume fraction (1–5%) of nanoparticles (NPs) of size of O(1–100 nm). The base fluids can be water and organic fluids such as ethanol and ethylene glycol. The NPs can be the oxides of aluminum and silicon, as well as metals such as copper and gold [49,50]. NFs allow for substantial enhancement in conductive heat transfer, as much as 40% increase in thermal conductivity [49,51,52], despite the low volume fraction of the NPs. On the other hand, conventional particle-liquid suspensions require high concentrations (>10%) of particles to achieve such enhancement. In practice, rheological and stability problems precluded the widespread application of high concentration suspensions. This makes NFs a valuable fluid for different industrial applications, especially in processes where cooling is of primary concern. Recently, there is an increasing focus on the convective properties of NFs in the literature. The presence of convection term is expected to lead to complex physical behavior [53]. One advantage that a fluid containing