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The onset of convection in a porous layer with multiple horizontal solid partitions

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ABSTRACT

The principal objective of the present paper is to investigate the onset of convection in a horizontal layer heated from below which consists of distinct porous sublayers which are separated by solid heat-conducting partitions. Each of the porous sublayers are identical as are the solid partitions. The present analysis employs linearised stability theory and a dispersion relation is derived from which neutral curves may be computed. For two-layer configurations the dispersion relation may be written explicitly, but for larger numbers of sublayers a simple systematic numerical procedure is used to compute the dispersion relation which, while it may also be written analytically, rapidly becomes increasingly lengthy as the number of sublayers increases. It is found that neutral curves are always unimodal and each has a well-defined single minimum. We attempt to give a comprehensive physical understanding of the effect of the number of layer, the relative thickness of the partitions and the conductivity ratio on the onset of convection and the form taken by the onset modes. Our results are compared with those of Rees and Genç (2011) [1] who considered the special case where the partitions are infinitesimally thin.

1. Introduction

A large number of papers have been published which have considered the effect of layering in one form or another on the onset and subsequent development of convection in layers heated from below. One of these, which considers two horizontal layers of fluid heated from below where the two fluid layers are separated by an impermeable horizontal interface, was written by Proctor and Jones [2]. A linear stability analysis yielded information about the onset of convection. It was found in some cases that the neutral stability curve is bimodal, and the authors then continued to consider weakly nonlinear convection where the two critical wavenumbers were in the ratio of 1:2. Catton and Lienhard V [3] considered a similar configuration but allowed the solid partition to be of finite thickness and therefore its conductivity became of importance.

In the present paper we also concentrate on a layer consisting of a number of sublayers, but attention is focussed on convection taking place in a porous medium rather than a clear fluid. The sublayers are identical in every respect and the partitions are also identical in every respect. Layering has been quite a favoured topic of study in the field of porous media because of its supposed application to geological systems. A series of papers by McKibbin and colleagues ([4–8] as well as others [9–11]) have teased out quite a substantial amount of information about the surprisingly detailed problem of the osnet of convection and its weakly nonlinear development. For a general layered system McKibbin and O'Sullivan [4] provided quite a comprehensive analysis of the onset problem and this was subsequently developed into a weakly nonlinear analysis by McKibbin and O'Sullivan [5]. A three-dimensional weakly nonlinear analysis by Rees and Riley [11] showed that two-dimensional convection is sometimes unstable, the realised pattern being a set of cells with square planform. They also found bimodal curves, which suggests that these may be quite ubiquitous in layered configurations. One example of a trimodal configuration was also found.

McKibbin and Tyvand [7] considered alternating configurations of sublayers, where neighbouring sublayers were thick and thin. The thermal properties of each type of sublayer were taken to be identical but the thin layers had low permeability. This meant that an anistropic modelling such as was undertaken in McKibbin and Tyvand [6] could not be done so easily because convection cells were found to be localised in the thick sublayers. Jang and Tsai [12] considered a three-layer configuration where the middle sublayer is impermeable, but thermally conducting, and of finite thickness. It was found that the system is at its most stable condition when the partition is located centrally. Rees and Genç [1] considered a variation on this overall theme by insisting that the thin layers were of infinitesimal thickness and impermeable. In such

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Nomenclature

A, B, C, D	constants
A^*, B^*	constants
CHF	constant heat flux
CT	constant temperature
d	conductivity ratio
$\underline{d}$	vector defined in (30)
g	gravity
h	height of the solid partitions
H	height of the porous sublayers
$\mathcal{H}$	height of the compsite layer
k	disturbance wavenumber
k_s	thermal conductivity of solid
k_{pm}	thermal conductivity of porous medium
K	permeability
$\mathcal{M}, \mathcal{N}$	4×4 matrices
N	number of porous sublayers
p	pressure
Ra	Darcy–Rayleigh number
t	time
T	temperature of solid
$\mathcal{T}$	disturbance in T
u	horizontal velocity
$\underline{v}$	vector of coefficients
w	vertical velocity
x	horizontal coordinate
z	vertical coordinate

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
γ	constant
δ	thickness ratio
ΔT	temperature scaled
θ	temperature of porous medium
Θ	disturbance in θ
κ	diffusivity ratio
λ	constant
μ	dynamic viscosity
ρ	density
σ	constant
ψ	streamfunction
Ψ	disturbance streamfunction

Subscripts and superscripts

c	cold boundary
h	hot boundary
j	sublayer index
pm	porous medium
s	solid phase
$'$	derivative with respect to z
1,2,...	sublayer indices

composite layers convection patterns are localised within the porous sublayers. There were three surprising results which were found: (i) neutral curves naturally bunch into groups of N when there are N sublayers; (ii) the dispersion relation for N sublayers factorises into N similar factors, and this facilitates a large part of the general analysis while explaining the bunching of the neutral curves, and (iii) the system tends towards one with a critical Rayleigh number of 12 and wavenumber of 0 as the number of sublayers increases — this is significant because these critical values correspond to a single porous layer subject to constant heat flux boundary conditions, whereas the overall problem has constant temperature boundary conditions.

In this paper we consider a more physically realistic version of the work undertaken by Rees and Genç [1]. The porous layer will consist of N identical porous sublayers, which are separated by identical solid partitions, but these partitions have finite thickness. A formula for the dispersion relation is obtained, account being taken of the temperature variations within the solid partitions. We obtain neutral curves, mode shapes and the manner of the variation in the critical values as the governing parameters (namely the diffusivity ratio, d, the thickness ratio, δ, and the number of porous sublayers, N) vary.

2. Governing equations

We consider the onset of convection in a horizontal porous layer which consists of a number of identical porous sublayers of thickness, H, which are separated by solid partitions each of thickness, h. Thus while fluid may not pass from one sublayer to another, conductive heat transfer may take place through the solid partitions. A configuration which consists of three sublayers is depicted in Fig. 1.

It is assumed that the Boussinesq approximation is valid, that the porous medium is homogeneous and isotropic, that the phases are in local thermal equilibrium, and that the fluid motion satisfies Darcy's law in addition to the buoyancy effects. Given the above dimensions, a general system of N porous sublayers has overall height, $\mathcal{H}$, given by

$$\mathcal{H} = NH + (N-1)h. \tag{1}$$

Dirichlet boundary conditions for temperature are applied on the outer horizontal surfaces of the layer, as shown in Fig. 1, and the continuity of both temperature and heat flux conditions are applied at all interfaces. The governing equations are non-dimensionalised by using H as the representative lengthscale, rather than $\mathcal{H}$, and by using the temperature drop across one sublayer, rather than across the system as whole; this has the advantage of yielding much easier comparisons between cases which consist of different numbers of sublayers, particularly the classical single-layer Darcy–Bénard problem. Given that we are performing a linear stability analysis in an unbounded horizontal layer, all three-dimensional modes may be decomposed into sums or integrals of two-dimensional roll cells. Here, we present our analysis in terms of two-dimensional equations. The non-dimensional governing equations for the problem considered herein are given by (see [13]),

$$\frac{\partial u_j}{\partial x} + \frac{\partial w_j}{\partial z} = 0, \tag{2}$$

$$u_j = -\frac{\partial p_j}{\partial x}, \tag{3}$$

$$w_j = -\frac{\partial p_j}{\partial z} + Ra\ \theta_j, \tag{4}$$

$$\frac{\partial \theta_j}{\partial t} + u_j \frac{\partial \theta_j}{\partial x} + w_j \frac{\partial \theta_j}{\partial z} = \frac{\partial^2 \theta_j}{\partial x^2} + \frac{\partial^2 \theta_j}{\partial z^2} \tag{5}$$

for porous sublayer j, where $1 \leqslant j \leqslant N$, and by

$$\frac{\partial T_j}{\partial t} = \kappa \left(\frac{\partial^2 T_j}{\partial x^2} + \frac{\partial^2 T_j}{\partial z^2} \right) \tag{6}$$

for solid layer, j where $1 \leqslant j \leqslant N-1$, and where κ is the diffusivity ratio, α_s/α_{pm}. In the above, Ra is the Darcy–Rayleigh number which is defined according by