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Development of the concept of mean temperatures in the analysis of radiative heat transfer in an inhomogeneous non-isothermal non-gray medium

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ABSTRACT

Concepts of an emission mean temperature and a transmission mean temperature are introduced and shown to be effective and necessary in characterizing the radiative emission and transmission of an inhomogeneous non-isothermal non-gray medium. Based on a two zone model for a $CO_2/H_2O/N_2$ mixture, the mathematical behavior of these two mean temperatures and their dependence on the temperature and the partial pressure of the absorbing gases of the two zones are illustrated. Results show that the average temperature is not appropriate to be used in the prediction of radiative emission and transmission in practical engineering systems.

1. Introduction

The importance of radiation heat transfer in many practical engineering systems and the related problems such as combustion furnaces and enclosure fires is well known. Historically, the implementation of radiative heat transfer in practical engineering calculation is limited because of the mathematical complexity in accounting for the spectral and geometric effect of radiation. Over the past thirty years, a significant amount of research has been conducted and sophisticated solution methods have been developed. To account for the geometric effect, for example, there are the zonal method [1–3], the discrete ordinate method [4–6] and many others. To account for the spectral effect, there are the narrow band model [7], the *k*-distribution method [8] and the weighted gray gas method [9,10]. In recent years, methods are also developed to account for both the spectral and geometric effect simultaneously [11]. Indeed, with the increasing availability of fast computational power and large data storage capability, many practical engineering CFD (computational fluid dynamic) codes such as FLUENT [12] and FDS [13] have now integrated a radiation heat transfer solver using one of the solution methods. Highly sophisticated CFD simulations with impressive graphic results accounting for the effect of radiation heat transfer now appear to be available and these codes are being used more frequently and routinely in important engineering design and fire safety analysis.

Fundamentally, while an accurate solution method which can account for the spectral and geometric dependence of radiation heat transfer is important, the effect of the radiative and thermal properties of the participating medium is equally important and can have a significant impact on the accuracy of the results. All practical engineering systems are generally inhomogeneous and non-isothermal. Most, if not all, of the solution methods for radiation heat transfer, however, are developed under the assumption of an isothermal, homogenous medium. In the implementation of these methods in a practical engineering calculation, some "average" properties are generally used in the solver. Since the radiation effect is non-localized, the determination of the appropriate "average" properties used in the radiation solver is therefore a key to an accurate simulation of radiative heat transfer in practical engineering systems. Indeed, the presentation of results generated by CFD codes without a detailed explanation on how the "average" radiation properties are evaluated is meaningless since the accuracy of the results can be highly uncertain.

Over the past 30 years, the area of radiative heat transfer in inhomogeneous and non-isothermal medium has not received much attention. Indeed, the only reported research efforts on the analysis of radiative heat transfer in inhomogeneous non-isothermal gases appear to be those on the development of the Curtis-Godson approximation [14–18] in the 1960s. These efforts, however, were focused on obtaining appropriate "equivalent uniform" properties for individual absorption band. Since different "equivalent" properties are determined for different absorption bands, this

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Nomenclature

a_λ	absorption coefficient
d_1	length scale of zone 1 (m), Fig. 1
d_2	length scale of zone 2 (m), Fig. 1
d_m	length scale of the equivalent homogeneous isothermal zone (m), Fig. 1
$e_{\lambda b}$	Planck function
P_{H_2O}	partial pressure of H_2O (kPa)
P_{CO_2}	partial pressure of CO_2 (kPa)
$Q_{ih,e}$	radiative heat flux emitted from the inhomogeneous mixture from the surface of zone 1
$Q_{h,e}$	radiative heat flux emitted from the equivalent homogeneous mixture
T	temperature (K)
$T_{m,em}$	emission mean temperature (K)
$T_{m,tr}$	transmission mean temperature (K)
τ_{ih}	transmissivity of the inhomogeneous mixture
τ_h	transmissivity of the equivalent homogeneous mixture

Subscripts

1, 2	label for two zones of the inhomogeneous mixture
m	label for the equivalent homogeneous zone
λ	wavelength

concept is not feasible for the determination of total heat transfer in a practical engineering system.

The objective of this work is to introduce the concept of two mean temperatures and the associated mean partial pressures for an inhomogeneous non-isothermal medium so that existing radiation solvers can be used accurately and efficiently in practical engineering calculation. Similar to the concept used by the Curtis-Godson approximation, the “equivalent” mean temperature and partial pressure are defined as the properties such that the total transmission through this “equivalent isothermal gas” and its total emission are identical to those of a given path consisting of an inhomogeneous non-isothermal gas. In this work, this concept is illustrated by a two-zone $CO_2/H_2O/N_2$ mixture. The numerical data are generated using an existing well established narrow-band radiation solver RADCAL [7]. The two mean temperatures are tabulated numerically for different gas properties and compositions to illustrate their dependence on the mixture properties. Based on these data, the error of the common approach which uses a volumetric average temperature in predicting the radiation effect is assessed.

2. Mathematical formulation

A two-zone non-isothermal inhomogeneous medium as shown in Fig. 1 will be used as a basis for the development of the “radiation mean” temperature. The medium is assumed to be a $CO_2/H_2O/N_2$ mixture at atmospheric pressure. The physical dimension for each zone will be assumed to be 1 m. For a given set of conditions (T_1, $P_{CO_2,1}d_1$, $P_{H_2O,1}d_1$, T_2, $P_{CO_2,2}d_2$, $P_{H_2O,2}d_2$) of the two-zone mixture, a mean temperature will be determined by finding the temperature of an equivalent isothermal, homogeneous medium as shown in Fig. 1, with conditions T_m, $P_{CO_2,m}d_m$, $P_{H_2O,m}d_m$, which yield the same radiative heat transfer result as the original mixture. For simplicity, the current work will consider only mixtures maintained at atmospheric pressure. Extension to higher pressure is straight forward and can be considered in future works.

The “equivalent” mixture is assumed to be at atmospheric pressure. The total mass of the absorbing gases, CO_2 and H_2O, will be assumed to be equal to those of the original two zones. Based on the mean temperature T_m and treating the mixture as an ideal gas, the partial pressure and the physical dimension of the equivalent isothermal and homogeneous medium is given by

$$d_m = T_m\left(\frac{d_1}{T_1}+\frac{d_2}{T_2}\right) \tag{1}$$

$$P_{H_2O,m}d_m = T_m\left(\frac{P_{H_2O,1}d_1}{T_1}+\frac{P_{H_2O,2}d_2}{T_2}\right) \tag{2}$$

$$P_{CO_2,m}d_m = T_m\left(\frac{P_{CO_2,1}d_1}{T_1}+\frac{P_{CO_2,2}d_2}{T_2}\right) \tag{3}$$

Since a different integration is involved in the evaluation of the emission and absorption of the mixture, a different mean temperature will be introduced for the two processes.

For the evaluation of emission from the two zone mixture radiating from the surface of the first zone as shown in Fig. 1, the total emission is given by

$$\begin{aligned} Q_{ih,e} = {} & \int_0^\infty e_{\lambda b}(T_1)\left\{1-\exp\left[-a_\lambda(T_1,P_{H_2O,1},P_{CO_2,1})d_1\right]\right\}d\lambda \\ & + \int_0^\infty e_{\lambda b}(T_2)\exp\left[-a_\lambda(T_1,P_{H_2O,1},P_{CO_2,1})d_1\right] \\ & \times\left\{1-\exp\left[-a_\lambda(T_2,P_{H_2O,2},P_{CO_2,2})d_2\right]\right\}d\lambda \end{aligned} \tag{4}$$

The emission for the equivalent one-zone homogenous medium is

$$Q_{h,e} = \int_0^\infty e_{\lambda b}(T_{m,em})\left\{1-\exp\left[-a_\lambda(T_{m,em},P_{H_2O,m},P_{CO_2,m})d_m\right]\right\}d\lambda \tag{5}$$

Eqs. (4) and (5) can be evaluated with any spectral integration method (e.g. line by line, narrow band model, etc.). By setting $Q_{ih,e} = Q_{h,e}$, a mean emission temperature, $T_{m,em}$, can be readily tabulated as a function of the mixture properties. Note that the concept of a mixture emissivity is meaningless for a non-isothermal mixture since it is a function of the assumed mixture temperature. The emission mean temperature is therefore determined based on the equivalence of the total radiative emission, not on the equivalence of the mixture emissivity.

For the evaluation of transmission through the two zone mixture, the following expression for the transmissivity is evaluated

$$\begin{aligned} \tau_{ih} = {} & \int_0^\infty e_{\lambda b}(T_s) \\ & \times\exp\left[-a_\lambda(T_1,P_{H_2O,1},P_{CO_2,1})d_1 - a_\lambda(T_2,P_{H_2O,2},P_{CO_2,2})d_2\right]d\lambda \end{aligned} \tag{6}$$

For the one-zone homogenous medium, the transmissivity is

$$\tau_h = \int_0^\infty e_{\lambda b}(T_S)\exp\left[-a_\lambda(T_{m,tr},P_{H_2O,m},P_{CO_2,m})d_m\right]d\lambda \tag{7}$$

By setting $\tau_{ih} = \tau_h$, a mean transmission temperature, $T_{m,tr}$, can be tabulated. Note that $T_{m,tr}$ is generally a function of both the mixture properties and the source temperature.

The two different mean temperatures can be readily evaluated using any spectral radiation solver. The current work uses the narrow band model, RADCAL [7], which has been shown to be sufficiently accurate for application in combustion media at atmospheric conditions.