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Heat and mass transfer on the peristaltic transport of non-Newtonian fluid with creeping flow

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ABSTRACT

A new mathematical model has been developed for the peristaltic transport of Maxwell fluid with heat and mass transfer, while taking into account the effect of thermal diffusion (Soret), occurring in an asymmetric channel with creeping flow. The inertia terms are omitted from the equations of motion, which leads to solutions that approximately valid for low Reynolds number, i.e. $Re \ll 1$. The walls are kept at different but constant temperatures and concentrations. A perturbation solution is acquired, which satisfies the momentum, energy and concentration equations for the case by choosing a small wave number. Numerical results are evaluated for pressure rise and frictional forces per wavelength. The velocity, temperature and concentration fields have been appraised for diverse values of the parameters entering into the problem. The influence of diverse parameters of interest on pumping, trapping, temperature and concentration profiles has been investigated graphically.

1. Introduction

Peristaltic pumping is a form of physiological fluid transport that occurs in the human body. Peristaltic action is an inherent neromuscular property of any tubular smooth muscle structure. The study of heat and mass transfer on the peristaltic transport has several industrial applications. Typical applications include processes such as drying, evaporation at the surface of a water body, energy transfer in wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. When heat and mass transfer occur simultaneously in a moving fluid, the relationship between the fluxes and the driving potentials is of more intricate nature. Mass fluxes can be created by temperature gradient which is also known as the Soret or thermal-diffusion effect. The Soret effect has been used for isotope separation and in mixture between gases with very light molecular weight (H_2,He) and of medium molecular weight (H_2,air) Gupta and Gupta [1]. In many previous studies the Soret effect has been neglected, on the basis that it is of a smaller order of magnitude than the effects described by Fick's laws: Radhakrishnamacharya and Radhakrishnamacharya [2], Chamkha and Khaled [3], Khan et al. [4], Postelnicu [5], Mekheimer [6] and Ogulu [7].

Mathematical models have already been derived for a train of periodic sinusoidal wave in an infinite or finite two dimensional symmetric or axi-symmetric channel/tubes containing Newtonian or non-Newtonian fluids. In this direction first initiative was taken by Latham [8]. Burns and Parkes [9] have investigated the peristaltic flow through axially symmetrical pipes and channel under the effect of creeping flow. Barton and Raynor [10] have investigated the peristaltic flow in tubes with approximation of low Reynolds number. Shapiro et al. [11] has also used the long wave length approximation in studying the peristaltic pumping phenomenon. Jaffrin and Shapiro [12] have investigated peristaltic pumping with neglecting inertia terms. Gupta and Seshadri [13] have analyzed the peristaltic flow through non-uniform channels and tubes with reference to the flow of spermatic fluid in vas deferens, neglecting the inertia terms. Srivastava and Srivastava [14] have extended the analysis of peristaltic transport to two layered model. Mekheimer [6] has studied the peristaltic transport of a viscous fluid (creeping flow) through the gap between coaxial tubes, where the outer tube is non uniform and has a sinusoidal wave traveling down its wall and the inner one is rigid, uniform tube and moving with a constant velocity.

Further, several authors have attempted to solve momentum equation related to peristaltic flows with diverse approximations along with associated heat and mass transfer. However, the creeping flows (low Reynolds number) have not been considered in the aforementioned work.

Major recent research related to the peristaltic transport with heat and mass transfer for Newtonian and non-Newtonian fluid includes the following works. Radhakrishnamacharya and Srinivasulu [15] have examined the influence of wall properties on peristaltic transport with heat transfer. Srinivas and Kothandapani [16] have done peristaltic transport in an asymmetric channel with

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heat transfer. Another approach consists of the studying the effect of heat transfer and magnetic induction on the viscous fluid in a vertical annulus Mekheimer and Elmabound [17]. Eldabe et al. [18] has studied the problem of peristaltic transport of a non-Newtonian fluid with variable viscosity in the presence of heat and mass transfer along with mixed diffusion flow between a vertical wall that deforms the shape of a travelling wave and a parallel flat wall. Srinivas and Kothandapani [19] have investigated the effects of heat and mass transfer on peristaltic transport in a porous space with compliant walls. Srinivas and Gayathri [20] have reported the peristaltic flow of viscous fluid in an asymmetric channel with heat transfer and porous medium.

Nadeem and Akbar [21] has discussed the influence of radially varying MHD on the peristaltic flow in an annulus with heat and mass transfer. Srinivas et al. [22] have investigated mixed convective heat and mass transfer in an asymmetric channel with peristalsis. El-Sayed et al. [23] has studied the effect of mass diffusion of chemical species on peristaltic transport through the vertical porous media in the gap between concentric tubes with heat and mass transfer. El-Sayed et al. [24] has also been studied magnetothermodynamic peristaltic flow of Bingham non-Newtonian fluid in eccentric annuli with slip velocity and temperature jump conditions. Tripathi and Beg [25] have studied the unsteady physiological magneto-fluid flow and heat transfer through a finite length channel by peristaltic pumping.

Through an extensive survey of existing literature one can say that creeping effect for non-Newtonian fluids for the peristaltic transport have not come under considerable attention. In contrast to existing work, this paper considers heat and mass transfer on peristaltic transport of creeping flow. The objective of the present work is to extend the flow analysis of peristaltic mechanism of Maxwell fluid in an asymmetric channel as presented in Hayat et al. [26] to creeping flow with Soret effect. The novelty of this work is that after applying the creeping flow assumption to Maxwell fluid, the effect of non-Newtonian fluid are incorporated. In this investigation, a mathematical model is presented to understand the influence of creeping flow on the peristaltic transport of Maxwell fluid in an asymmetric channel with heat and mass transfer. The momentum, energy and concentration equations are simplified by neglecting inertia terms and analytic solutions for the flow variables have been derived. The features of the flow characteristics are analyzed by plotting graphs and discussed in details. The contributions of Soret number in particular, and those of the geometrical parameters in general, to the flow, heat and mass transfer characteristics are found to be quite significant and interesting.

The rest of this paper has been organized as follows: Mathematical model has been presented in Section 2, problem has been formulated in Section 3, Perturbation based solution has been presented in Section 4 and detailed discussion about the results has been given in Section 5. Finally, some conclusions have been drawn out in Section 6.

2. Mathematical model

The motion of the steady, heat and mass transfer equations of Maxwell fluid is governed by the following system of equations:

$$\nabla.\mathbf{V} = 0, \tag{1}$$

$$\rho(\mathbf{V}.\nabla)\mathbf{V} = \nabla\mathbf{T}' + \rho\mathbf{b} + \mathbf{R}, \tag{2}$$

$$\mathbf{T}'.\mathbf{L} - \nabla.\mathbf{q} + \rho\mathbf{r} = \rho(\mathbf{V}.\nabla)e, \tag{3}$$

$$(\mathbf{V}.\nabla)C = D.\nabla(\nabla C) + \frac{DK_T}{T_m}\nabla.(\nabla.\mathbf{T}) - k_1 C, \tag{4}$$

where

$$\mathbf{T}' = -p\mathbf{I} + \mathbf{S}, \tag{5}$$

$$\mathbf{S} + \Lambda_1\left(\frac{d\mathbf{S}}{dt} - \mathbf{LS} - \mathbf{SL}\right) = \mu\mathbf{A}_1, \tag{6}$$

$$\mathbf{L} = \nabla.\mathbf{V}, \tag{7}$$

$$\mathbf{A}_1 = (\nabla.\mathbf{V}) + (\nabla.\mathbf{V})^t. \tag{8}$$

Here, $\mathbf{V}$ is the velocity vector, $\mathbf{b}$ is the body force (assumed to be zero), $\mathbf{R}$ is the Darcy's resistance, p is the pressure, μ is the constant viscosity, Λ_1 is the relaxation time, ρ is the fluid density, $\mathbf{S}$ is the extra stress tensor, T' is the Cauchy stress tensor, $\mathbf{A}_1$ is the first Rivlin–Ericksen tensor, $\mathbf{r}$ is the radiant heating (assumed to be zero), $e = C_pT$ is the specific internal energy, where C_p is the specific heat and T is the temperature, $\mathbf{q} = -k\nabla T$ is the heat flux vector, where k is the constant thermal conductivity, C is the mass concentration, T_m is the mean fluid temperature, K_T is the thermal diffusion ratio and k_1 is the chemical reaction parameter.

3. Formulation of the problem

Let us consider the steady and incompressible flow of Maxwell fluid in an asymmetric channel. The surface is maintained at uniform constant temperature and concentration, see Fig. 1. The flow has significant Soret effect while satisfying creeping flow assumption.

The motion of an incompressible fluid is caused by sinusoidal wave trains propagating with constant speed c along the channel walls as defined by the following pair of equations:

$$h_1'(X',t') = d_1 + a_1\sin\left(\frac{2\pi}{\lambda_1}(X' - ct')\right),$$
$$h_2'(X',t') = -d_2 - b_1\sin\left(\frac{2\pi}{\lambda_1}(X' - ct') + \varphi\right). \tag{9}$$

In Eq. (9), a_1 and b_1 are the waves amplitude, $d_1 + d_2$ is the channel width, λ_1 is the wave length, c is the wave speed, $\varphi(0 \leqslant \varphi \leqslant \pi)$ is the phase difference, X' and Y' are the rectangular coordinates. Moreover, $\varphi = 0$ corresponds to symmetric channel with waves out of phase and for $\varphi = \pi$ the waves are in phase. Further, a_1, b_1, d_1, d_2 and φ satisfy the condition: $a_1^2 + b_1^2 + 2a_1b_1\cos\varphi \leqslant (d_1 + d_2)^2$. The wall $Y = h_1'$ is kept at a temperature T_0 and concentration C_0 and the wall $Y = h_2'$ is kept at a temperature T_1 and concentration C_1.

Introducing a wave frame (x', y') moving with velocity c away from the fixed frame (X', Y') by the transformation:

$$x' = X' - ct, \quad y' = Y', \quad u'(x', y') = U'(X', Y', t') - c, \quad v'(x', y') = V'(X', Y', t'), \tag{10}$$

where (u', v') are the velocity components in wave frame. The governing equations in the wave frame are given as below:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0, \tag{11}$$

$$\rho\left[u'\frac{\partial}{\partial x'} + v'\frac{\partial}{\partial y'}\right]u' = -\frac{\partial p'}{\partial x'} + \frac{\partial S'_{x'x'}}{\partial x'} + \frac{\partial S'_{x'y'}}{\partial y'}, \tag{12}$$

$$\rho\left[u'\frac{\partial}{\partial x'} + v'\frac{\partial}{\partial y'}\right]v' = -\frac{\partial p'}{\partial y'} + \frac{\partial S'_{x'y'}}{\partial x'} + \frac{\partial S'_{y'y'}}{\partial y'}, \tag{13}$$

$$\rho C_p\left[u'\frac{\partial}{\partial x'} + v'\frac{\partial}{\partial y'}\right]T = k\left[\frac{\partial^2 T}{\partial x'^2} + \frac{\partial^2 T}{\partial y'^2}\right] + S'_{x'x'}\frac{\partial u'}{\partial x'} + S'_{y'y'}\frac{\partial v'}{\partial y'} + S'_{x'y'}\left[\frac{\partial v'}{\partial x'} + \frac{\partial u'}{\partial y'}\right] - \frac{1}{\rho}\frac{\partial q_r}{\partial y}, \tag{14}$$