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Linear and weakly nonlinear triple diffusive convection in a couple stress fluid layer

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ABSTRACT

The effect of couple stresses on linear and weakly nonlinear stability of a triply diffusive fluid layer is investigated. Several departures not observed either in singly or doubly diffusive couple stress fluid layer have been identified while analyzing the linear stability of the problem. In contrast to the doubly diffusive couple stress fluid system, oscillatory convection is found to occur even if the diffusivity ratios are greater than unity. The presence of couple stress is to increase the threshold value of solute Rayleigh number beyond which oscillatory convection is preferred. Moreover, disconnected closed oscillatory neutral curves are identified for certain choices of physical parameters indicating the requirement of three critical values of Rayleigh number to specify the linear stability criteria instead of the usual single value. Besides, heart-shaped oscillatory neutral curves are also found to occur in some cases and the effect of couple stress parameter on some of these unusual behaviors is analyzed. A weakly nonlinear stability analysis is performed using modified perturbation technique and the stability of steady bifurcating non-trivial equilibrium solution is discussed. Heat and mass transfer are calculated in terms of Nusselt numbers and the influence of various physical parameters on the same is discussed in detail.

1. Introduction

Double diffusive convection is characterized by well mixed convecting layers and occurs if gradients of two competing stratifying agencies (heat and salt or any two solute concentrations) having different diffusivities are simultaneously present. One of the most interesting aspects of double diffusive instabilities is that even stabilizing overall density gradient can destabilize the system when the density gradients caused by individual components are opposed. It is observed that when the two individual diffusing components are opposed, salt fingers occur when the component with the smaller diffusivity is destabilizing, while oscillatory convection occurs when the faster diffusing component is destabilizing. The subject has attracted considerable interest over the last few decades and the significant developments took over have been largely due to its relevance and applications in many fields such as oceanography, astrophysics, geophysics and engineering. Copious literature is available on double diffusive convection and the topic has been reviewed extensively [1–7].

However, fluid dynamical systems cited above provide many examples of convective phenomena in which the density depends on three or more stratifying agencies having different molecular diffusivities. Thus one can expect multicomponent convection. As a first step towards understanding multicomponent convection, knowledge about how a triple diffusive system behaves differently from those of double diffusive systems is warranted. This is because, with the addition of a more slowly diffusing property to the bottom layer of a double diffusive system that would otherwise have produced a finger interface could cause a diffusive interface to form. Similarly, addition of the same property to the top layer of another system may change the resulting interface from a diffusive to a salt finger kind. The possibilities of existing of such interesting situations have prompted researchers to study convective instability in triple diffusive fluid systems both theoretically and experimentally.

Griffiths [8] was the first to investigate theoretically the linear stability of triple diffusive convection in a horizontal fluid layer, while Griffiths [9,10] and Turner [3] reported the related experimental works. Coriell et al. [11] and Noulty and Leaist [12] presented explicit situations in which triple diffusive convection has practical significance. Pearlstein et al. [13] performed a detailed study on the linear stability of a triply diffusive fluid layer. They have completely captured the physics of the onset of convection and showed that the triple diffusive system is capable of supporting several remarkable departures from what occurs in the singly and doubly diffusive fluid systems which were overlooked previously. Terrones and Pearlstein [14] generalized the linear stability

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Nomenclature

C_i (i = 1,2) solute concentration of the ith component
d depth of couple stress fluid layer
$\vec{g}$ acceleration due to gravity
$\hat{k}$ unit vector in the z-direction
ℓ, m wave numbers in the x and y directions
p pressure
$Pr = \nu/\kappa_t$ Prandtl number
$R_t = \beta_t g d^3 \Delta T/\nu\kappa_t$ thermal Rayleigh number
$R_{si} = \beta_{ci} g d^3 \Delta C_i/\nu\kappa_t$ (i = 1,2) solute Rayleigh number of the ith component
$\vec{q} = (u, v, w)$ velocity vector
t time
T temperature
(x,y,z) Cartesian co-ordinates

Greek symbols
α horizontal wave number
β_t thermal expansion coefficient
β_{ci} (i = 1,2) solute analog of β_t for the ith component
σ growth rate
κ_t thermal diffusivity
κ_{c1}, κ_{c2} solute analogs of κ_t
μ dynamic viscosity
μ_c couple stress viscosity
$\nu = \mu/\rho$ kinematic viscosity
$\psi(x,z,t)$ stream function
$\Lambda_c = \mu_c/\mu d^2$ couple stress parameter
ρ fluid density
ρ_0 reference density
$\tau_i = \kappa_{ci}/\kappa_t$ (i = 1,2) ratio of diffusivities of the ith component

analysis to an arbitrary number of components in a horizontal fluid layer. In the context of nonlinear stability analysis, Moroz [15] considered the linear stability problem originally discussed by Griffiths [8]. Lopez et al. [16] revealed the effect of rigid boundaries on convective instability in a triply diffusive fluid layer. The effects of cross-diffusion on the onset of convective instability in a horizontally unbounded triply cross-diffusive fluid layer have been investigated by Terrones [17]. Straughan and Walker [18] analyzed various aspects of penetrative convection in a triply diffusive fluid layer, while multicomponent convection – diffusion with internal heating or cooling in a fluid layer is considered by Straughan and Tracey [19].

All the aforementioned investigations on triple diffusive convection have been dealt with Newtonian fluids. In the study of many triply diffusive fluid dynamical problems mentioned above, the hypothesis of a Newtonian fluid will be too restrictive and cannot precisely describe the characteristics of the fluid flow involved therein. Therefore, probing the problems considering non-Newtonian effects are quite desirable and appropriate. Unlike Newtonian fluids, there are different kinds of non-Newtonian fluids and obviously they do not lend themselves to a unified treatment. In particular, polar fluids have received wider attention in recent years. These fluids deform and produce a spin field due to the microrotation of suspended particles. As far as these types of non-Newtonian fluids are concerned, there are two important theories proposed by Eringen [20] and Stokes [21] and these are, respectively, referred to as micropolar fluid theory and couple stress fluid theory. The micropolar fluids take care of local effects arising from microstructure and as well as the intrinsic motions of microfluidics. The spin field due to microrotation of freely suspended particles setsup an antisymetric stress, known as couple stress, and thus forming couple stress fluid. The couple-stress fluid theory represents the simplest generalization of the classical viscous fluid theory that allows for polar effects and whose microstructure is mechanically significant in fluids. For such a special kind of non-Newtonian fluids, the constitutive equations are given by Stokes [21] which allows the sustenance of couple stresses in addition to usual stresses. This fluid theory shows all the important features and effects of couple stresses and results in equations that are similar to Navier–Stokes equations. Couple-stress fluids have applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plates in a bath, exotic lubricants and colloidal fluids, electro-rheological fluids to mention a few.

Based on the formulation of Stokes [21], convective instability in a singly and doubly diffusive couple-stress fluid layer has been investigated in the recent past. Malashetty and Basavaraja [22] investigated the onset of Rayleigh–Benard convection in a layer of couple stress fluid under the influence of thermal/gravity modulation. The linear and non-linear double diffusive convection with Soret effect in couple stress liquids has been considered by Malashetty et al. [23], while Gaikwad et al. [24] reported the results on linear and non-linear double diffusive convection by considering both Soret and Dufour effects in couple stress liquids.

The effect of couple stresses on single and double diffusive fluid systems is considered in the past. Nonetheless, many fluid dynamical systems occurring in nature and engineering applications entail three or more diffusing components. Examples include molten polymers, salt solutions, slurries, geothermally heated lakes, magmas and their laboratory models, synthesis of chemical compounds and so on in which the fluid flow involved can be well characterized by couple stress fluid theory rather than Newtonian relationship. Moreover, there is all possibility of displaying variety of behavior by the fluid dynamical system, not observed in singly and doubly diffusive couple stress fluid systems, with three or more diffusing components. Under the circumstances, it is of interest to gain a general understanding of the manner in which the presence couple stresses affects the convective instability of a triply diffusive fluid layer. The main objective of the present study is therefore to investigate the effects of couple stresses on the linear and weakly nonlinear stability of a triply diffusive fluid layer and uncover the presence of couple stresses on some of the unusual behaviors of the system under certain conditions and also on the heat and mass transport.

2. Mathematical formulation

We consider an initially quiescent horizontal incompressible couple stress fluid layer of thickness d in which the density depends on three stratifying agencies namely, temperature T as well as solute concentrations C_1 and C_2 having different diffusivities. The density is assumed constant everywhere except in the body force and the off-diagonal contributions to the fluxes of the stratifying agencies are neglected. A Cartesian coordinate system (x,y,z) is used with the origin at the bottom of the fluid layer and the z-axis vertically upward. The gravity is acting vertically downwards with the constant acceleration, $\vec{g} = -g\hat{k}$ where $\hat{k}$ is the unit vector