



Technical Note

Thermal instability in a porous medium layer saturated by a nanofluid: A revised model

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ABSTRACT

We develop an extension of our previous thermal instability analysis of a nanofluid-saturated porous layer. The extension is based on a new boundary condition for the nanoparticle fraction, which is physically more realistic. In the previous model we imposed both temperature and nanoparticle volume fractions at the boundaries of the layer. It is now assumed that the value of the temperature can be imposed on the boundaries, but the nanoparticle fraction adjusts so that the nanoparticle flux is zero on the boundaries. The new boundary condition on the nanoparticle volume fraction is made possible by accounting for the contributions of the effect of thermophoresis to the nanoparticle flux. It is shown that, with the new boundary conditions, oscillatory convection cannot occur. The effect of the nanoparticles on non-oscillatory convection is destabilizing.

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1. Introduction

The problem of the onset of convection in a horizontal layer occupied by a porous medium uniformly heated from below is a classical problem commonly known as the Horton–Rogers–Lapwood problem or Bénard–Darcy problem. Work on this problem has been surveyed in Chapter 6 of the book by Nield and Bejan [1].

An extension to the case of a nanofluid, based on a model presented by Buongiorno [2], in which the effects of Brownian motion and thermophoresis are taken into account, was made by Nield and Kuznetsov [3]. In their paper it was assumed that one could control the value of the nanoparticle fraction at the boundary in the same way as the temperature there could be controlled. However, since the development of the model presented in [3], it became apparent that in practice controlling the nanoparticle volume fraction on the boundaries may be difficult. Thus it is advisable to replace the boundary conditions used in [3] by a set that are more realistic physically. In the present paper we revisit this problem, and we now assume that there is no nanoparticle flux at the plate and that the particle fraction value there adjusts accordingly. This change has important consequences. The scaling of dimensionless parameters needs changing. The basic solution for the nanoparticle volume fraction is changed, and conclusions are changed. No longer are there two opposing agencies affecting instability and hence oscillatory instability is ruled out.

We have submitted a companion paper revising our paper on the corresponding problem in a fluid clear of solid material [4].

2. Analysis

The analysis follows closely that in [3] and so it is abbreviated as far as possible here.

We select a coordinate frame in which the z^* -axis is aligned vertically upwards. We consider a horizontal layer of a porous medium of porosity ε and permeability K confined between the planes $z^* = 0$ and $z^* = H$. Asterisks are used to denote dimensional variables. Each boundary wall is assumed to be impermeable and perfectly thermally conducting. The temperatures at the lower and upper walls are taken to be T_h^* and T_c^* , respectively, T_h^* being greater than T_c^* . For simplicity, Darcy's law is assumed to hold and the Oberbeck–Boussinesq approximation is employed. Homogeneity and local thermal equilibrium in the porous medium is assumed. The reference temperature is taken to be T_c^* . In the linear theory being applied here the temperature change in the fluid is assumed to be small in comparison with T_c^* . The Darcy velocity is denoted by \mathbf{v}^* . The field equations for total mass, momentum, thermal energy, and nanoparticles, respectively, take the form

$$\nabla^* \cdot \mathbf{v}^* = 0, \quad (1)$$

$$0 = -\nabla^* p^* - \frac{\mu}{K} \mathbf{v}^* + \left[\phi^* \rho_p + (1 - \phi^*) \{ \rho (1 - \beta (T^* - T_c^*)) \} \right] \mathbf{g}, \quad (2)$$

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Nomenclature

c	nanofluid specific heat at constant pressure
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
H	dimensional layer depth
k_m	effective thermal conductivity of the porous medium
Le	Lewis number, defined by Eq. (15)
N_A	modified diffusivity ratio, defined by Eq. (19)
N_B	modified particle-density increment, defined by Eq. (20)
p^*	pressure
p	dimensionless pressure, $p^*K/\mu\alpha_m$
Ra	thermal Rayleigh–Darcy number, defined by Eq. (16)
Rm	basic-density Rayleigh number, defined by Eq. (17)
Rn	concentration Rayleigh number, defined by Eq. (18)
t^*	time
t	dimensionless time, $t^*\alpha_m/\sigma H^2$
T^*	nanofluid temperature
T	dimensionless temperature, $\frac{T^* - T_c^*}{T_h^* - T_c^*}$
T_c^*	temperature at the upper wall
T_h^*	temperature at the lower wall
(u, v, w)	dimensionless Darcy velocity components, $(u^*, v^*, w^*)H/\alpha_m$
\mathbf{v}	Darcy velocity, $\varepsilon\mathbf{v}$
\mathbf{v}^*	dimensional Darcy velocity, (u^*, v^*, w^*)

(x, y, z) dimensionless Cartesian coordinates, $(x^*, y^*, z^*)/H$; z is the vertically-upward coordinate
 (x^*, y^*, z^*) Cartesian coordinates

Greek symbols

α	dimensionless wavenumber
α_m	thermal diffusivity of the porous medium, $\frac{k_m}{(\rho c_p)_f}$
μ	viscosity of the fluid
ρ	fluid density
$(\rho c)_m$	effective heat capacity of the porous medium
ρ_p	nanoparticle mass density
σ	parameter defined by Eq. (8)
ϕ^*	nanoparticle volume fraction
ϕ_0^*	reference value for nanoparticle volume fraction
ϕ	relative nanoparticle volume fraction, $\frac{\phi^* - \phi_0^*}{\phi_0^*}$

Superscripts

*	dimensional variable
'	perturbation variable

Subscript

b	basic solution
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$$(\rho c)_m \frac{\partial T^*}{\partial t^*} + (\rho c)_f \mathbf{v}^* \cdot \nabla^* T^* = k_m \nabla^{*2} T^* + \varepsilon (\rho c)_p [D_B \nabla^* \phi^* \cdot \nabla^* T^* + (D_T/T_c^*) \nabla^* T^* \cdot \nabla^* T^*], \quad (3)$$

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\varepsilon} \mathbf{v}^* \cdot \nabla^* \phi^* = D_B \nabla^{*2} \phi^* + (D_T/T_c^*) \nabla^{*2} T^*. \quad (4)$$

We write $\mathbf{v}^* = (u^*, v^*, w^*)$. We assume that the temperature is constant and the nanoparticle flux is zero on the boundaries. Thus the boundary conditions are

$$w^* = 0, \quad T^* = T_h^*, \quad D_B \frac{\partial \phi^*}{\partial z^*} + \frac{D_B}{T_\infty} \frac{\partial T^*}{\partial z^*} = 0 \quad \text{at } z^* = 0, \quad (5)$$

$$w^* = 0, \quad T^* = T_c^*, \quad D_B \frac{\partial \phi^*}{\partial z^*} + \frac{D_B}{T_\infty} \frac{\partial T^*}{\partial z^*} = 0 \quad \text{at } z^* = H. \quad (6)$$

We introduce dimensionless variables as follows. We define

$$(x, y, z) = (x^*, y^*, z^*)/H, \quad t = t^* \alpha_m / \sigma H^2, \\ (u, v, w) = (u^*, v^*, w^*)H/\alpha_m, \\ p = p^* K / \mu \alpha_m, \quad \phi = \frac{\phi^* - \phi_0^*}{\phi_0^*}, \quad T = \frac{T^* - T_c^*}{T_h^* - T_c^*}, \quad (7)$$

where ϕ_0^* is a reference scale for the nanoparticle fraction and

$$\alpha_m = \frac{k_m}{(\rho c_p)_f}, \quad \sigma = \frac{(\rho c_p)_m}{(\rho c_p)_f}. \quad (8)$$

Then Eqs. (1)–(6) take the form:

$$\nabla \cdot \mathbf{v} = 0, \quad (9)$$

$$0 = -\nabla p - \mathbf{v} - Rm \hat{\mathbf{e}}_z + Ra T \hat{\mathbf{e}}_z - Rn \phi \hat{\mathbf{e}}_z, \quad (10)$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T + \frac{N_B}{Le} \nabla \phi \cdot \nabla T + \frac{N_A N_B}{Le} \nabla T \cdot \nabla T, \quad (11)$$

$$\frac{1}{\sigma} \frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} \mathbf{v} \cdot \nabla \phi = \frac{1}{Le} \nabla^2 \phi + \frac{N_A}{Le} \nabla^2 T, \quad (12)$$

$$w = 0, \quad T = 1, \quad \frac{\partial \phi}{\partial z} + N_A \frac{\partial T}{\partial z} = 0 \quad \text{at } z = 0, \quad (13)$$

$$w = 0, \quad T = 0, \quad \frac{\partial \phi}{\partial z} + N_A \frac{\partial T}{\partial z} = 0 \quad \text{at } z = 1. \quad (14)$$

Here

$$Le = \frac{\alpha_m}{D_B}, \quad (15)$$

$$Ra = \frac{\rho g \beta K H (T_h^* - T_c^*)}{\mu \alpha_m}, \quad (16)$$

$$Rm = \frac{[\rho_p \phi_1^* + \rho(1 - \phi_1^*)] g K H}{\mu \alpha_m}, \quad (17)$$

$$Rn = \frac{(\rho_p - \rho) \phi_0^* g K H}{\mu \alpha_m}, \quad (18)$$

$$N_A = \frac{D_T (T_h^* - T_c^*)}{D_B T_c^* \phi_0^*}, \quad (19)$$

$$N_B = \frac{\varepsilon (\rho c)_p}{(\rho c)_f} \phi_0^*. \quad (20)$$

The parameter Le is a Lewis number and Ra is the familiar thermal Rayleigh–Darcy number. The parameters Rm and Rn may be regarded as a basic-density Rayleigh number and a concentration Rayleigh number, respectively. The parameter N_A is a modified diffusivity ratio and is somewhat similar to the Soret parameter that arises in cross-diffusion phenomena in solutions, while N_B is a modified particle-density increment.

By extending the Oberbeck–Boussinesq approximation, Eq. (10) has been linearized by the neglect of a term proportional to the product of ϕ and T . We believe that this assumption will be valid in the case of small temperature gradients in a dilute suspension of nanoparticles.

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