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Technical Note

A novel effective medium theory for modelling the thermal conductivity of porous materials

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ABSTRACT

A novel effective medium theory was proposed to model the thermal conductivity of porous materials. In this theory, phases (or components) are treated as small spheres dispersing into an assumed uniform medium with the thermal conductivity k_m. A simple algebraic expression for the thermal conductivity based on this theory was derived, in which each has a distinct physical basis. The expression can unify five basic structural models (Series, Parallel, two forms of Maxwell–Eucken, effective medium theory) through variations of k_m. Furthermore, the feasibility of the model was evaluated using the experimental data from previous literatures and those calculated by this model.

1. Introduction

Nowadays there are heat attentions on porous materials due to their widespread industrial applications. The thermal conductivity plays an important role in the performance of these materials. From the point of view of heat conduction, porous materials can be considered as a two-phase (or two-component) system, viz. a solid skeleton and air, and thermal conductivity can be used to describe heat transfer through this complex system [1]. In this case, there are many analytical models proposed to predict the thermal conductivity of porous materials [2,3].

The thermal conductivity of porous materials is a complex property, as it is not only dependent on the properties of the solid component and porosity but also the structure of the materials [4]. For simple structures five basic structural models including the Series and Parallel models [5], Maxwell–Eucken models [6], effective medium theory (EMT) equation [7] were proposed. Afterwards some models for slightly complex structures such as the area contact model [8] and unit cell method [9] were constructed. However, each of these models assumes a certain physical structure and could not be applicable to all types of structure. Instead, a common approach is adding an empirical weighting parameter in basic structural models to account for differences in structure [10]. But the parameter is determined experimentally without any distinct physical basis, and cannot reflect the actual structure of the porous materials.

In this paper, a procedure for modelling the thermal conductivity of porous materials using a novel effective medium theory was presented. Five basic structural models can be unified by a simple equation without any empirical parameter.

2. Theory

Five basic structural models mentioned-above (shown schematically in Fig. 1), including the Series, Parallel, Maxwell–Eucken (two forms) and EMT models, are shown below in respective order for a two-component system [11]:

$$k_e = \frac{1}{(1-v_2)/k_1 + v_2/k_2} \tag{1}$$

$$k_e = k_1(1-v_2) + k_2 v_2 \tag{2}$$

$$k_e = k_1 \frac{2k_1 + k_2 - 2(k_1 - k_2)v_2}{2k_1 + k_2 + (k_1 - k_2)v_2} \quad \text{(component 1 continuous)} \tag{3}$$

$$k_e = k_2 \frac{2k_2 + k_1 - 2(k_2 - k_1)(1-v_2)}{2k_2 + k_1 + (k_2 - k_1)(1-v_2)} \quad \text{(component 2 continuous)} \tag{4}$$

$$(1-v_2)\frac{k_1 - k_e}{k_1 + 2k_e} + v_2 \frac{k_2 - k_e}{k_2 + 2k_e} = 0 \tag{5}$$

where k and v are thermal conductivity and volume fraction, and subscripts of e, 1 and 2 represent the two-component material system, components 1 and 2, respectively.

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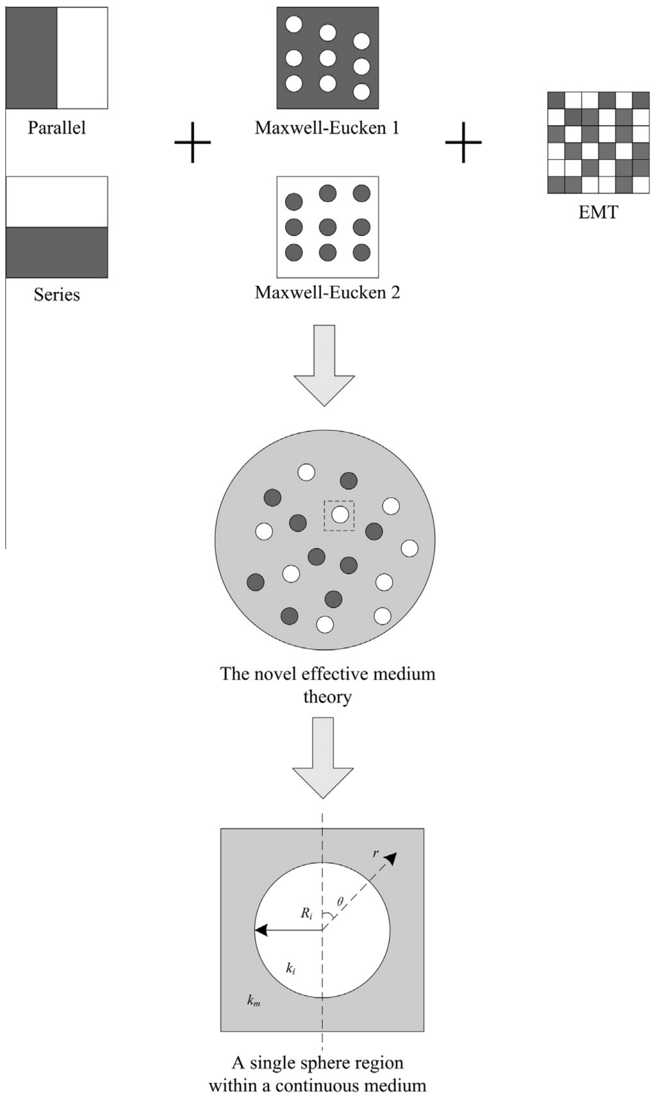

Fig. 1. Schematic illustrations of the novel effective medium theory.

In order to make these basic structural models more generic for different structures, an extra parameter is sometimes introduced. For example, Krischer proposed a weighted harmonic mean of the Series and Parallel models, where the weighting parameter f (sometimes referred to as the 'distribution factor') has a value ranging between 0 and 1 [10]:

$$k_e = \frac{1}{\left[\frac{1-f}{k_1(1-v_2)+k_2 v_2} + f\left(\frac{1-v_2}{k_1} + \frac{v_2}{k_2}\right)\right]}. \tag{6}$$

Similarly, Maxwell–Eucken and EMT models have been modified by Hamilton and Kirkpatrick respectively, as follows:

$$k_e = k_1 \frac{(f-1)k_1 + k_2 - (f-1)(k_1-k_2)v_2}{(f-1)k_1 + k_2 + (k_1-k_2)v_2} \tag{7}$$

and

$$(1-v_2)\frac{k_1-k_e}{k_1+(f/2-1)k_e} + v_2\frac{k_2-k_e}{k_2+(f/2-1)k_e} = 0. \tag{8}$$

In principle, the modified models such as Eqs. (6)–(8), may be used to predict the thermal conductivity of any structure with a suitable value of f, as long as the data lie within the envelope bounded by the Series and Parallel models. Although these modified models have been widely used, the weighting parameter f is determined empirically without any distinct physical basis.

In this paper, we propose a novel effective medium theory for multi-phase (or multi-component) materials. Fig. 1 shows the schematic illustrations of the novel effective medium theory in present work. The primary assumption is that phases (or components) are treated as small spheres dispersing into an assumed uniform medium with the thermal conductivity k_m. If a material consists of i phases, phases i is considered as n small spheres of radius R_i and thermal conductivity k_i contained within an uniform medium with thermal conductivity k_m.

For a single small sphere region within a uniform medium under steady-state conditions, the temperature distribution is governed by Laplace's Equation, as shown below in two-dimensional polar coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) = 0 \tag{9}$$

following boundary conditions:

- at $r = 0 \quad T_i \neq \infty$;
- at $r = R_i \quad k_i\frac{\partial T_i}{\partial r} = k_m\frac{\partial T_m}{\partial r}$ and $\frac{\partial T_i}{\partial\theta} = \frac{\partial T_m}{\partial\theta}$;
- at $r \gg R_i \quad T_m = br\cos\theta$

where, r and θ are polar radius and polar angle in polar coordinate, T is the temperature, the constant b (K/m) is the magnitude of the temperature gradient in the continuous medium, and subscript of i and m represent phase i and assumed uniform medium. A general solution of Eq. (9) is:

$$T = A + \frac{B}{r} + Cr\cos\theta + \frac{D}{r^2}\cos\theta \tag{10}$$

Using the boundary conditions to substitute for A, B, C and D in Eq. (10) yields:

$$T_i = b\frac{3k_m}{k_i+k_m}r\cos\theta \quad \text{(within the sphere)} \tag{11}$$

and

$$T_m = br\cos\theta - bR_i^3\frac{k_i-k_m}{k_i+2k_m}\frac{\cos\theta}{r^2} \quad \text{(outside the sphere)}. \tag{12}$$

Because of n small spheres for each phase, Eq. (12) should become:

$$T_m = br\cos\theta - \Sigma bnR_i^3\frac{k_i-k_m}{k_i+2k_m}\frac{\cos\theta}{r^2}. \tag{13}$$

On the other hand, the material could be considered as a large sphere of radius R and thermal conductivity k_e contained within the uniform medium. The volume fraction v_i of phase i within the material is:

$$v_i = \frac{nR_i^3}{R^3}. \tag{14}$$

So, Eq. (12) is:

$$T_m = br\cos\theta - \sum bv_iR^3\frac{k_i-k_m}{k_i+2k_m}\frac{\cos\theta}{r^2}. \tag{15}$$

Because the large sphere with thermal conductivity k_e is filled within the assumed uniform medium, Eq. (12) would become:

$$T_m = br\cos\theta - bR^3\frac{k_e-k_m}{k_e+2k_m}\frac{\cos\theta}{r^2}. \tag{16}$$

Eq. (15) should be equal to Eq. (16), and hence we get:

$$\Sigma v_i\frac{k_i-k_m}{k_i+2k_m} = \frac{k_e-k_m}{k_e+2k_m}. \tag{17}$$