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Benchmark data on laminar Rayleigh–Bénard convection in a stratified supercritical fluid: A case of two-dimensional flow in a square cell

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ABSTRACT

We simulate numerically steady-state Rayleigh–Bénard convection slightly above the stability threshold in a stratified supercritical fluid. We use a two-dimensional approximation and consider one roll contained in a square cell. Simulations are performed by two different numerical codes. First code is based on the complete Navier–Stokes equations, and second one is a low Mach number approximation with filtering sound and stratification effects. We consider conditions in which an influence of adiabatic temperature gradient is significant. Solutions found on the basis of two mathematical models are matched in a special fashion to find a contribution of adiabatic temperature gradient and specify common features independent of whether the governing equations are able to predict a stratification effect. We perform a comparison with known analytical and experimental results and propose our solutions as benchmark data for supercritical Rayleigh–Bénard convection.

1. Introduction

Fluids near the gas–liquid critical point display distinctive physical properties, particularly the specific heat at constant pressure, the isothermal compressibility, and the thermal expansion coefficient grow unboundedly approaching to the critical point [1,2]. We consider homogeneous media at the temperature slightly exceeding the critical one which are usually called near- or supercritical fluids. Specific physical properties lead to strong thermomechanical coupling and significant peculiarities in dynamic and thermal behavior of such fluids. Extensive theoretical investigations of motions and heat transfer near the critical point on the basis of thermohydrodynamical models start from [3–6] giving an analysis of fast thermalization from a heat source inside a confined domain; this phenomenon is associated with adiabatic heating and called the piston effect. During more than two decades the great attention addresses on thermal gravity-driven convection which develops near the critical point almost in all real conditions. This reason leads to that a study of convective processes is very important. The investigations clarified some special features of supercritical convection which are associated with intensification of convective motions, interplay with the piston effect, and an influence of density stratification. First circumstance means that convective motions become more and more intensive approaching to the critical point due to a growth of the thermal expansion coefficient. Second one gives rise to convection develops in the temperature field disturbed in many cases by the piston effect. The piston effect is resulted from the very high isothermal compressibility and is considerable in confined cells under unsteady heat supply. In steady regimes, the piston effect is negligible. Abnormal large compressibility leads also to a significant role of density stratification that is a growth of fluid density in direction of the gravity force due to the own weight of fluid. In normal fluids, density stratification becomes significant only in large geometrical scales, for example in atmosphere. In supercritical fluids, due to the very high isothermal compressibility, the density variation in several percents can occur in laboratory scales [7].

Our investigation focuses on a dynamic behavior of supercritical fluids slightly above the stability threshold in the Rayleigh–Bénard configuration (horizontal layer heated from below). In such configuration, some temperature gradient co-directing to the gravity force can be applied to a layer without hydrodynamic instabilities to develop. In a classical incompressible fluid, the onset of convection is controlled by the Rayleigh criterion which is associated with viscous and thermal dissipations. If a fluid is compressible, it is subjected to density stratification. Rising, an element of fluid expands and cools adiabatically because the density and pressure decrease with the height. If the temperature gradient applied to the fluid layer is less than the adiabatic temperature gradient (ATG), a convective motion does not develop [8]. This condition related to an adiabatic process is known as the Schwarzschild stability criterion. The onset of convection regarding both dissipative and adiabatic stabilising mechanisms was firstly analyzed for a perfect gas

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Nomenclature

$\boldsymbol{B}$	perfect gas constant normalized by the molecular weight (J kg^{-1} K^{-1}), R/μ_g
c_p	specific heat at constant pressure (J kg^{-1} K^{-1})
c_v	specific heat at constant volume (J kg^{-1} K^{-1})
$\dot{\mathrm{D}}$	strain rate tensor (s^{-1})
Fr	Froude number
$\mathbf{g}$	acceleration of gravity (m s^{-2})
$\boldsymbol{J}$	heat flux at boundary in a convective fluid (W m^{-2})
J_d	heat flux at boundary in a motionless fluid (W m^{-2})
k	stratification coefficient, $1-\Gamma_a/\Gamma$
K	kinetic energy (kg m^2 s^{-2})
l	side of cell (m)
M	Mach number
Nu	Nusselt number
p	pressure component defined as a difference between P and $\langle P\rangle$ (Pa)
P	total pressure (Pa)
$\langle P\rangle$	volume-average pressure (Pa)
Pr	real Prandtl number
Pr_0	model Prandtl number
$\mathbf{r}=(x,y)$	radius-vector (m)
R	perfect gas constant (J mol^{-1} K^{-1})
Ra	real Rayleigh number
Ra_0	model Rayleigh number
ra^*	threshold Rayleigh number for an unlimited horizontal layer, 1.708×10^3
t	time (s)
T	temperature (K)
$\mathbf{U}=(u,v)$	velocity vector (m s^{-1})
$\boldsymbol{\alpha}_p$	thermal expansion coefficient (K^{-1})
γ	ratio of specific heats
$\boldsymbol{\Gamma}$	applied temperature gradient (K m^{-1})
$\boldsymbol{\Gamma}_a$	adiabatic temperature gradient (ATG) (K m^{-1})
ε	temperature distance to the critical point, $(T-T_c)/T_c$
ς	bulk viscosity (Pa s)
η	dynamic viscosity (Pa s)
$\boldsymbol{\Theta}$	temperature difference responsible for convection (K)
$\boldsymbol{\Theta}_{Ra}$	Rayleigh temperature difference (defined at $k=1$) (K)
$\boldsymbol{\Theta}_{Sc}$	Schwarzschild temperature difference (defined at $k=0$) (K)
λ	thermal conductivity (W m^{-1} K^{-1})
μ_g	molecular weight (kg mol^{-1})
ρ	density (kg m^{-3})

Superscripts

s	value modified by including a stratification effect
'	dimensional value
$*$	threshold value

Subscripts

c	critical point
i	initial condition
0	perfect gas

[9]. The similar problem in the case of supercritical fluids was solved in [10] and, later, in [11,12]; the piston effect does not influence the onset of convection [13]. In [14] conditions on a convection onset and corrected expression of ATG in the van der Waals fluid were found.

The high-precise experiments on convective heat transport in supercritical ^{3}He inside a cylinder cell at bottom heating [15,16] have triggered series of numerical studies of unsteady Rayleigh–Bénard convection in high compressible fluids. Several research groups report on an overshoot and a subsequent damped oscillation of the temperature difference at horizontal boundaries in the stage transient toward steady state [17–24]. In these studies, following conditions of experiments, a constant heat flux at the bottom with a fixed top temperature are applied. Corrections of numerical data for the ATG effect allow one to reduce the results on heat transfer to universal relation [21–23]. Different scenarios for starting Rayleigh–Bénard convection at the same boundary conditions are described in [25]. Development of convective motions at the fixed temperature difference at boundaries is investigated numerically as well. As simulated in [26], heating from below leads to boundary layers at the bottom and top to form so that instabilities grow in these two different layers. In [27–29], a competition between two limits defined by Rayleigh and Schwarzschild criteria is studied. As shown, this competition leads to the reverse transition to stability through the Schwarzschild line. The effect of criticality diverging bulk viscosity on the convective structure is investigated in [30].

Steady-state convection regimes reaching in a long time are simulated to study stabilizing effect of ATG [31], to display universal features of heat transfer in near-critical fluids [21–23,32–34]. The experiments and simulations show that the convection heat current versus the reduced Rayleigh number collapses onto a single curve [18,21–23,32–35]. The similarity theory developed in [32–34] for gravity-driven convection in the van der Waals fluid allows one to determine the real Rayleigh and Prandtl numbers near the critical point. Simulated convection in a near-critical fluid is compared with that in a perfect gas at the same real Rayleigh and Prandtl numbers. In [36] an intriguing stabilizing effect of ATG at a dominant role of Schwarzschild criterion is found. If the applied temperature gradient is very close to ATG, a layer of fluid can be stabilized approaching to the critical point or transiting to a stronger gravity.

To simulate a variety of complex thermohydrodynamic effects near the critical point, one should employ adequate mathematical models. The Boussinesq and low Mach number approximations [37] often applied to noncritical convection are not able to predict density stratification and, consequently, an influence of ATG on the convection onset and convective motions. The acoustic filtering procedure in these approximations breaks the dependence of density on the pressure component responsible for isothermal compressibility. Therefore the low Mach number approximation is employed only if the role of ATG is negligible [23,24,26]. To simulate hydrostatic effects, the low Mach number approximation is adapted accounting for relatively strong stratification [38] (this approximation was applied later in [27,29]) or hydrodynamic models similar to an extended Boussinesq approximation are specially developed for supercritical fluids [17,18]. The complete Navier–Stokes equations are used as well [30] however numerical simulations in this case require a lot of computational expenses because of very short time step to be determined by a time of sound propagation. To reduce the time of computations, the complete Navier–Stokes equations are modified by pressure decomposition without acoustic filtering. This modification firstly applied in [39,40] to describe noncritical dynamical problems includes two-scale splitting of the pressure and a replacement of pressure gradient in the moment equation. The complete Navier–Stokes equations modified in this manner allow one to carry out simulations at a large time step despite sound is not excluded. The similar model is used in [32–34]