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Mass transfer during radial oscillations of gas bubbles in viscoelastic mediums under acoustic excitation

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ABSTRACT

The generation of acoustic cavitation in viscoelastic mediums (e.g. human or animal tissue) is an essential topic for facilitating non-invasive therapeutic ultrasonic treatment of serious diseases (e.g. tumors). In present paper, mass transfer during radial oscillations of gas bubbles in viscoelastic mediums under acoustic excitation is theoretically investigated and influences of several parameters (e.g. shear modulus, saturation condition and viscosity) on the mass diffusion across bubble interfaces are discussed. The characteristics of acoustic cavitation generated *in vivo* are also explained based on our predictions by re-visiting the pioneering studies in the field. Comparing with previous predictions in the literature, our predictions reveal that values of maximum bubble sizes growing through mass diffusion is larger and required time reaching above maximum bubble size is longer, suggesting that medium viscoelasticity is one of paramount parameters for predicting mass diffusion of cavitation bubbles in tissue.

1. Introduction

When gas bubbles in mediums are irradiated by acoustic waves, gas bubbles will oscillate owing to the pressure changes in the mediums caused by the acoustic excitation. During oscillations of gas bubbles, not only the interface area and volume of gas bubbles but also pressure and gas concentration inside gas bubbles change. Furthermore, the oscillating gas bubbles can also generate radial fluid flow within surrounding mediums, which will influence gas diffusion across bubble interface according to the gas diffusion equation. Therefore, the bubbles will grow or dissolve under acoustic excitation, termed as "rectified mass diffusion", which has been investigated both theoretically and experimentally by many researchers over several decades [1–35]. For reviews of this topic, readers are referred to [10,17]. Rectified mass diffusion serves as a paramount mechanism in many physical (e.g. bubble sonoluminescence [22–24]), chemical (e.g. sonochemistry [27]) and biomedical (e.g. transdermal transport of molecules [28]) processes. Recently, generation of cavitation bubbles through rectified mass diffusion in viscoelastic mediums (e.g. human or animal tissues) is more and more involved such as cavitation assisted non-invasive therapeutic ultrasonic treatment of serious diseases (e.g. tumors). For some cases, encapsulated microbubbles are employed to strength cavitation activities [36]. For details of modeling of encapsulated microbubbles, readers are referred to Doinikov and Bouakaz [36].

For cavitation-effect non-invasive therapy, generation of cavitation through rectified mass diffusion in viscoelastic medium (e.g. human tissues) is usually involved [37]. Generation of acoustic cavitation *in vivo* through rectified mass diffusion has been demonstrated by many researchers both experimentally [14,15] and theoretically [9]. In a pioneer work by ter Haar and Daniels [14], a guinea-pig limb was irradiated using continuous ultrasound and a pulse echo ultrasonic imaging technique was used to visualize both moving and stationary bubbles of diameters down to 10 μm. Crum and Hansen [9] proposed that rectified mass diffusion is the primary mechanism for the bubble growth under irradiation of ultrasound *in vivo* observed by ter Haar and Daniels [14] and ter Haar et al. [15]. In Crum and Hansen [9], the non-Newtonian properties of the tissue of the limb tested in [14] were neglected and the limb tissue was assumed as Newtonian fluids with surface tension coefficient modified. For some recent studies using Crum–Hansen approach, readers are referred to Lavon et al. [28]. Crum and Hansen [9] further noticed that the maximum bubble diameter and time required for the gas bubble growing to the maximum size are both far below those observed during experiments in [14]. Due to the highly complex nature of the problem, the reason for the above discrepancy is still not clear until now. In present paper, influences of viscoelastic effects of tissue on predictions of rectified mass diffusion are discussed.

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Nomenclature

Roman letters

c	concentration of the gas in the liquid
c_l	speed of sound in the liquid
C_0	saturation concentration of the gas in the liquid
C_i	initial uniform concentration of the gas in the liquid and also the concentration of the gas in the liquid at infinity
C_s	concentration of the gas in the liquid at the bubble wall
D	diffusion constant
$D_{g,v}$	thermal diffusivity of the gas defined at constant volume
f	frequency of the driving sound field
G	shear modulus
$k_{\boldsymbol{H}}$	Henry's constant
M_g	molecular weight of the gas in the bubble
P_0	ambient pressure
P_A	acoustic pressure amplitude
P_{in}	instantaneous pressure at the gas side of bubble wall
P_T	threshold of acoustic pressure amplitude of rectified diffusion
r	radial coordinate
R	instantaneous bubble radius
$\dot{R}$	first derivative of the instantaneous bubble radius
$\ddot{R}$	second derivative of the instantaneous bubble radius
R_0	equilibrium bubble radius
R_g	universal gas constant
t	time
T	period of applied acoustic excitation
T_∞	ambient temperature in the liquid
$\mathbf{u}$	velocity of the liquid
x	non-dimensional perturbation of the instantaneous bubble radius
$\dot{x}$	first time derivative of x
$\ddot{x}$	second time derivative of x

Greek letters

β_{ac}	acoustic damping constant
β_{th}	thermal damping constant
β_{tot}	total damping constant
β_{vis}	viscous damping constant
γ	ratio of specific heats of gas
γ_{rr}	strain
ε	non-dimensional amplitude of driving sound field
κ	polytropic exponent
μ_l	viscosity of the liquid
μ_{th}	effective thermal viscosity
ρ_g	density of the gas
ρ_l	density of the liquid
σ	surface tension coefficient
τ_{rr}	stress in the radial direction
φ	a function related with the solution of bubble interior problem
ω	angular frequency of the driving sound field
ω_0	natural frequency of oscillating gas bubbles
ω_r	resonance frequency corresponding to the maximum amplitude of steady-state gas bubble oscillations

Bubble dynamics in non-Newtonian mediums is a classic topic and has been intensively studied by many researchers [38–42]. Specifically, mass transfer across bubble interfaces in the absence of acoustic excitation has been well explored in the literature [43–49]. In this paper, a theoretical analysis of acoustically induced mass transfer across bubble interfaces in viscoelastic mediums (e.g. tissues) is presented. The influences of paramount parameters (e.g. viscoelasticity) on the gas bubble rectified mass diffusion are shown and discussed. Predictions based on present model with viscoelasticity are also compared with previous predictions [9] in the literature based on a model for rectified mass diffusion of gas bubbles in Newtonian mediums. The whole paper is organized as follows: Section 2 introduces the basic equations to be solved for the rectified mass diffusion phenomenon; Section 3 investigates the natural frequency and damping mechanisms during radial oscillations of gas bubbles in viscoelastic medium with numerical validations and demonstrating examples; Section 4 shows the analytical solution of rectified mass diffusion in viscoelastic mediums and influences of several paramount parameters on this phenomenon; Section 5 summarizes the concluding remarks of the present papers.

2. Basic equations

In this section, basic equations governing radial oscillations of gas bubbles in viscoelastic mediums under acoustic excitation will be introduced. Problems relating with radial oscillations of spherical gas bubbles oscillating in infinite viscoelastic mediums are to be solved. The relationship between the instantaneous volume and the inner pressure of the gas bubbles is described by the polytropic model. For convenience, energy dissipation through heat transfer across bubble interfaces is represented by an effective thermal viscosity. The equation of bubble motion is the generalized Keller-Miksis equation [50] such as

$$\left(1-\frac{\dot{R}}{c_l}\right)R\ddot{R}+\frac{3}{2}\left(1-\frac{\dot{R}}{3c_l}\right)\dot{R}^2=\left(1+\frac{\dot{R}}{c_l}\right)\frac{p_{ext}(R,t)-p_s(t)}{\rho_l}+\frac{R}{\rho_l c_l}\frac{d[p_{ext}(R,t)-p_s(t)]}{dt}, \tag{1}$$

where

$$p_{ext}(R,t)-p_s(t)=P_{in}-\frac{2\sigma}{R}-P_0-P_A\cos(\omega t)+3\int_R^\infty\frac{\tau_{rr}}{r}dr, \tag{2}$$

$$P_{in}=\left(P_0+\frac{2\sigma}{R_0}\right)(R_0/R)^{3\kappa}-\frac{4\mu_{th}\dot{R}}{R}. \tag{3}$$

Here, R is the instantaneous bubble radius; the overdot denotes the time derivative; c_l is the speed of sound in the liquid; ρ_l is the density of the liquid; t is the time; P_{in} is the instantaneous pressure at the gas side of bubble wall; σ is the surface tension coefficient; P_0 is the ambient pressure; P_A is the amplitude of the driving sound field; ω is the angular frequency of the driving sound field; r is the radial coordinate; τ_{rr} is the stress in the radial direction; R_0 is the equilibrium bubble radius; κ is the polytropic exponent; μ_{th} is the effective thermal viscosity. For convenience, the phase of external acoustic excitation (Eq. (2)) is ignored. For completeness, an effective thermal viscosity is further introduced in Eq. (3) by us based on [50] to consider energy dissipation through heat transfer across bubble interfaces. To close Eq. (2), a linear Voigt model is chosen as constitutive equation such as [50]

$$\tau_{rr}=2(G\gamma_{rr}+\mu_l\dot{\gamma}_{rr}), \tag{4}$$

with

$$\gamma_{rr}=-(2/3r^3)\left(R^3-R_0^3\right). \tag{5}$$

Here, G is the shear modulus; μ_l is the viscosity of the liquid; γ_{rr} is the strain. Combining Eqs. (2)–(5), one can obtain [50]