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A density variant drift flux model for density wave oscillations

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ABSTRACT

The present model emphasises on the development of an analytical model to describe density wave oscillation in a heated channel. The study includes linear stability analysis of density wave instability in vertical flow channels for high system pressure conditions. Unlike the previous models where the coolant density in single phase region was assumed to be constant (equal to saturated liquid density at given system pressure), the present work considers the density variation in the subcooled region. Drift flux model has been used to characterise the two-phase flow. The stability characteristics of the system have been depicted by stability boundaries plotted in parameter planes namely phase-change number (N_{pch}) and subcooling number (N_{sub}). The experimental data sets from Saha et al. (1974) [11] have been re-evaluated and are shown with corresponding operating conditions. Compared to the existing models where the equations run into several pages (Karve, 1998 [8]; Dokhane, 2004 [9]), it is seen that in the presence of linear–linear approximations for single phase enthalpy and two-phase quality that lead to simpler equations, the results obtained show good agreement with experimental data sets. In addition to higher subcooling numbers, the present model also best predicts the stability boundary for lower subcooling numbers.

1. Introduction

It is evident from past several years that two-phase flow systems such as nuclear reactors (BWR, AHWR etc.), steam generators and other similar kind of systems used in process or chemical industries are highly susceptible to flow oscillations. These oscillations are defined as instabilities. Among various kinds of such instabilities, density wave instability is a constituent of them [1,2]. Extensive studies have been carried out by researchers to analyse the characteristics of DWO followed by its corresponding controlling methods. Moreover, large system codes like RETRAN [3] and RAMONA [4] are used to pursue non-linear studies of the system. Those studies also include prediction of stability characteristics of the system. However, the study being expensive and time consuming produces the scope of developing reduced order models [5–7]. These reduced order models contain manageable number of equations through which the whole system dynamics can be captured.

In a continuation to the development of accurate models to study DWO in aforementioned systems, Achard et al. [5] developed a pair of functional delay differential equations (DDE) with complicated nonlinear multiple integral operators. However in this model, homogeneous equilibrium model (HEM) has been used to correlate the two-phase properties. Rizwan-uddin and Dorning [6] followed the model with drift flux equations leading to even more complicated DDEs. Later Clausse and Lahey [7] introduced the concept of time dependent spatial approximation of single phase enthalpy and two-phase quality, leading to comparably simplified set of coupled nonlinear equations. Following the similar approach, Karve [8] used quadratic approximations for single phase enthalpy and two-phase quality with HEM. This model helps to reduce the order of the systems as set of coupled nonlinear ordinary differential equations (ODEs) which are useful to carry bifurcation analysis. However, it is known that HEM may not be useful [9] to low flow rate conditions, hence the approach was extended by Dokhane [9,10] using drift flux model (DFM). It is worth noting that though these models predict the stability characteristics of the system having good agreement with experimental data sets [11], but due to the extensively large and tedious form of the set of coupled ODEs, make the analysis comparably difficult. Karve et al. [12] also presented a simple model using linear approximations for single phase enthalpy and two-phase quality. Though it reduces the complexity of the model, but it generates a significant amount of discrepancy between the theoretically obtained SB and experimental data sets [11].

In all the aforementioned studies, following the assumption of incompressible flow, the single phase density of the coolant is considered to be constant (saturation density). Moreover the stability boundaries are given in parameter planes of subcooling number (N_{sub}), and phase change number (N_{pch}). Due to the fact that density

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Nomenclature

A^*	cross section area of flow channel (m^2)
C_0	void distribution parameter
D_h^*	hydraulic diameter (m)
f	friction factor
Fr	Froude number ($\frac{v_0^{*2}}{g^* L_{ch}^*}$)
g^*	acceleration due to gravity (m/s^2)
h^*	enthalpy (kJ/kg)
J	Jacobian matrix
K_{inlet}	inlet pressure loss coefficient
K_{exit}	exit pressure loss coefficient
L_{ch}^*	total length of flow channel (m)
N_f	friction number ($\frac{fL_{ch}^*}{2D_h^*}$)
N_{pch}	phase change number ($\frac{q''^* \Delta\rho^* \xi_h^* L_{ch}^*}{A^* \Delta h_{fg}^* v_0^* \rho_g^* \rho_f^*}$)
N_{sub}	subcooling number ($\frac{(h_{sat}^* - h_{inlet}^*)\Delta\rho^*}{\Delta h_{fg}^* \rho_g^*}$)
q''^*	wall heat flux (W/m^2)
t^*	time (s)
v_0^*	reference velocity (m/s)
v_{inlet}^*	inlet velocity of coolant (m/s)
V_{gj}^*	average drift velocity (m/s)
z^*	distance along the axis of flow channel (m)
μ^*	boiling boundary (m)
ξ_h^*	heated perimeter (m)
ρ^*	density (kg/m^3)

Subscripts

1Φ	single phase
2Φ	double phase
f	liquid
g	vapour
m	mixture
ex	exit
in	inlet

Superscripts

$*$	dimensional quantity
$\sim$	steady state value

Abbreviations

AHWR	advanced heavy water reactor
BWR	boiling water reactor
DDE	delay differential equation
DFM	drift flux model
DWO	density wave oscillation
HEM	homogeneous equilibrium model
SB	stability boundary

of a subcooled liquid is a function of pressure and enthalpy in these kinds of isobaric systems, there arises discrepancy between the accounted density and actual density. Hence the density variation [13] of the liquid with gain in heat along the length of the coolant channel is taken into account in the present study. However, due to considerably low flow velocities (low Mach number), the incompressibility assumption of the flow still remains valid. Unlike the earlier model [13] the present model uses drift flux equations to correlate the two-phase properties.

Thus the main objective of the current research is to develop a simpler model for DWO while keeping a new insight to include the effect of density variation in single phase and drift between liquid and vapour phases in the two-phase region. The achievement of this objective is of practical interest, concerning the extent at which stability maps can be predicted closely to experimental data [11] as well as exact solutions [6] even in the presence of approximation techniques for single phase enthalpy and two-phase quality.

2. Mathematical model

As mentioned in earlier section, the present study is devoted towards the DWO in a heated channel. Density wave oscillation is a dominant type of dynamic instability [2] experienced by several two-phase flow systems. The consequences of dynamic instability can be interpreted as existence of sufficient interaction and delayed feedback between flow inertia and compressibility of two-phase mixture [2]. This results into multiple feedbacks between flow rate, pressure drop, change in density and consequently net vapour generation rate during boiling. The aim of this section is to simulate and analyse the aforementioned phenomenon in a heated channel.

To meet the purpose, the whole system is analysed by a 1-D axial flow model using three basic conservation PDEs of mass, energy and momentum along the length of the channel. These PDEs are reduced to corresponding ODEs, using weighted residual procedure [8]. Though in real situation the system dynamics is complex, but some simplifying assumptions are made [12] to develop the model that helps to simulate the aforementioned phenomenon.

Assumptions used in this model are as follows,

1. The system pressure remains constant.
2. Input heat flux is uniform along the length of the channel.
3. Boussinesq approximation is valid for variation of density in the two phase region.
4. Subcooled boiling is neglected.
5. $\partial p/\partial t$ term in energy conservation equation is neglected.

With these assumptions, the conservation equations of mass, momentum and energy for one-dimensional two-phase flow are given by:

Mass conservation

$$\frac{\partial}{\partial t^*}\{\rho^*\} + \frac{\partial}{\partial z^*}\{\rho^* v^*\} = 0 \tag{1}$$

Momentum conservation in single phase

$$\rho^*\left(\frac{\partial v^*}{\partial t^*} + v^*\frac{\partial v^*}{\partial z^*}\right) = -\frac{\partial P_{1\Phi}^*}{\partial z^*} - \frac{f^*}{2D_h^*}\rho^* v^{*2} - \rho^* g^* \tag{2a}$$

Momentum conservation in two-phase

$$\rho_m^*\left(\frac{\partial v_m^*}{\partial t^*} + v_m^*\frac{\partial v_m^*}{\partial z^*}\right) = -\frac{\partial P_{2\Phi}^*}{\partial z^*} - \frac{f_m^*}{2D_h^*}\rho_m^* v_m^{*2} - \rho_m^* g^* - \frac{\partial}{\partial z^*}\left(\frac{\alpha(z^*,t^*)}{1-\alpha(z^*,t^*)}\frac{\rho_g^*\rho_f^*}{\rho_m^*}V_{gj}^{*2}\right) \tag{2b}$$

Energy conservation

$$\frac{\partial}{\partial t^*}\{\rho^* h^*\} + \frac{\partial}{\partial z^*}\{\rho^* v^* h^*\} = \frac{q''^*\xi_h^*}{A^*} \tag{3}$$

As shown in Fig. 1, the channel is divided into two sections, single phase and two-phase. Similar to the earlier model [13], along with the density variation of subcooled water in the single phase, here also time dependent linear approximation of single phase enthalpy and two-phase quality have been taken into consideration.