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The influence of vibrations on the stability of thermocapillary flow in liquid zone

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ABSTRACT

The influence of high-frequency vibrations of one of supporting rods on the axisymmetrical thermocapillary flow in the liquid half-zone and its linear stability respective to three-dimensional perturbations in the framework of generalized Boussinesq approximation was investigated numerically. The generation of mean vorticity in dynamical boundary layers is taken into account using effective boundary conditions. The influence of amplitude of vibrations and Prandtl number at different Weber number values is considered. Calculations were carried out using the finite differences method. It is shown, that vibrations can be used to control the flow, to reduce its intensity and to stabilize it.

1. Introduction

One of the promising methods of obtaining high-quality semiconductor monocrystals is their growth from the melt by the floating zone method, which allows avoiding undesirable contact with the crucible walls. The distribution of dopant concentration in the crystal is determined by its distribution in the melt near the crystallization front. The concentration itself is influenced by convective transport as well as by diffusion. Thermogravitational convection in the melt, leading to the appearance of inhomogeneity in the dopant distribution and deteriorating the crystal structure, can be significantly reduced in the microgravity environment. However, thermocapillary convection takes place even in the absence of gravity being produced by the gradient of surface tension on the free surface of the melt. At large Marangoni number values thermocapillary convection becomes oscillatory, deteriorating the crystal quality. Therefore, the problem of determination of optimal conditions for the technological process and the problem of the convection control in the liquid zone arises. During the last thirty years, many studies were devoted to this problem. In order to control the convection a number of methods was offered, such as stationary or rotating magnetic fields of different configuration, rotation of the supporting rods. Another relatively recent method is application of vibrations.

The model configuration of the so-called half-zone, constituting capillary bridge between two disks of equal diameter, maintained at different temperature, is frequently used for experimental and numerical investigation of thermocapillary flows in the floating zone. The detailed experimental investigation of oscillatory instability of half-zone for the liquids with Pr = 1, 7 and 49 is accomplished in the work [1]. The dependencies of critical Marangoni number on the temperature difference and aspect ratio were investigated, as well as the spatial structure of over-critical oscillatory modes. The influence of gravity was studied by the comparison of results for zones, heated from above and from below.

A number of theoretical and numerical investigations were also dedicated to the stability of the flow in half-zone. Such methods as energy analysis [2], analysis of linear stability [3], direct three-dimensional numerical simulation [4] were used. The dependence of the stability threshold on the geometrical factor and Prandtl number was obtained [5,6]. It was shown that in the case of low ($Pr < 0.07$) and high ($0.5 < Pr < 5$) Prandtl numbers the instability is different in its nature. At intermediate Prandtl numbers the problem was studied in [7].

The flow, generated in the liquid bridge supported by two rigid surfaces with one of them vibrating in order to excite its capillary oscillations, was observed experimentally by Mollot et al. [8] and by Anilkumar et al. [9–12]. The oscillations of one of the supporting cylinders produce surface flow directed away from the vibrating wall and returning flow in the bulk of the liquid. The intensity of the flow increases with the frequency and the amplitude of vibrations as well as the length of the zone growth, and decreases with the growth of viscosity. This controllable surface flow was used to counterbalance the stationary thermocapillary flow of the opposite direction in the model half-zone of silicone oil. The temperature

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measurements in the zone show that radial gradient of temperature, produced by the thermocapillary flow, can be reduced by this counterbalancing.

An example of 3D simulation of thermocapillary flow in a liquid bridge can be found in [13].

In order to study the interaction of thermocapillary flow and the flow, produced by the low-frequency vibrations of supporting rods, the asymptotic model for the axisymmetrical half-zone in the approximation of low viscosity was developed in [14]. The numerically-analytical investigation of mechanisms of generation of vibrational flow in the liquid bridge of low viscosity was carried out in [15]. The numerical study of the possibility of suppression of thermocapillary flow in the half-zone and in the full zone, using vibrations of quite high frequency (supposing the small length of capillary waves on the free surface compared to the zone dimensions), was carried out in [16].

In order to study the behavior of the systems under the influence of high-frequency vibrations it is convenient to decompose the flow field into mean and pulsating components. This approach was used, e.g., by Longuet-Higgins [17] to describe mean flow under surface waves in the water. It was found that pulsating vorticity is significant in the narrow area near oscillating surfaces. Non-linear interaction leads to generation of mean vorticity which diffuses from the boundary layer into the bulk of the fluid due to viscosity. This effect, known as acoustic flow, is intensively studied recent years but in the majority of investigations only the case of liquid systems of infinite dimension is considered.

An additional thermovibrational volumetric effect appears when the liquid bridge is placed in non-isothermal conditions (which leads to the appearance of gradient of density in the volume of the liquid). The investigation of interaction of thermovibartional and thermocapillary flows was for the first time carried out in the work of Gershuni, Lyubimov, Lyubimova and Roux [18] in the frame of the model neglecting both vibrating and mean deformations of the free surface. The case, when both rigid cylinders supporting the liquid bridge are synchronously vibrating in the direction of the system axis and the bridge is heated up in the radial direction, was considered. It was found, that vibrations generate two-vortex vibrational flow and the direction of these vortices is opposed to the direction of two thermocapillary vortices situated in the same area. Consequently, thermovibrational flow can suppress the thermocapillary one. Besides, vibrational flow orients isotherms perpendicularly to the direction of vibrations, contributing to the flattening of the interface in the real technological processes of crystal growth.

Lyubimov [19] has shown that for the systems with free surfaces or interfaces it is fundamentally important to take into account vibrating deformations of free surface: they lead to the appearance of non-uniform vibrating velocity and generate mean vorticity in the viscous boundary layers near rigid walls and free surfaces (the effects of acoustic streaming). This author has also demonstrated that conventional Boussinesq approximation is not applicable in the case of non-uniformly heated fluid with deformable free surface in the presence of high-frequency vibrations. As well, he has developed a general method including a generalized Boussinesq approximation for weakly non-isothermal fluid and effective boundary conditions allowing taking into account pulsating deformations of free surface and effects of generations of acoustic streaming.

This approach was used for modeling the flow produced by high-frequency vibrations of both rods in isothermal and non-isothermal liquid bridge in [20,21], for studying the influence of high-frequency vibrations of one supporting rod on the flow in the isothermal axisymmetric undeformable and deformable liquid bridge in [22] and for investigation of the influence of high-frequency vibrations of the growing crystal on the linear stability of stationary axisymmetric soluto-thermocapillary flow in the liquid zone in [23].

This paper is devoted to the numerical investigation of high-frequency vibrations of one supporting rod on the stability of stationary axisymmetric thermocapillary flow in the liquid half-zone in the microgravity conditions.

2. Formulation of the problem

2.1. Governing equations

The liquid bridge between two parallel rigid disks of radius R, at the distance L from each other, and maintained at the temperatures T_0 and $T_0 + \Delta T$, is under consideration (Fig. 1). The less heated disk performs monochromatic vibrations in the direction of the symmetry axis with the frequency ω and the amplitude a (A is the vertical coordinate of the disk):

$$A = \mathrm{Re}\{-ia\exp(i\omega t)\} \tag{1}$$

The volume of the liquid is such that in the absence of the vibration the bridge has the shape of circular cylinder of radius R.

Let's suppose, that the vibration frequency is high, and the amplitude is low, so the following relations are satisfied:

$$(\nu/\omega)^{1/2} \ll R, \tag{2}$$

$$a \ll R. \tag{3}$$

Here ν is the kinematic viscosity. For the semiconductor melts its characteristic value is $\sim(1 \div 4)\cdot 10^{-3}\ \mathrm{cm/s^2}$, typical radius is $\sim 1 \div 0.5$ cm, which leads to the estimation $\omega \gg 0.016\ \mathrm{s^{-1}}$. So, the approach is applicable for vibration frequencies above order of units or tens Hz. An acceptable amplitude value is of order of 10^{-3} cm.

In this case it is convenient to decompose all the fields into slowly varying (mean) and quickly oscillating (vibrating) components and, using the many scales method, obtain for them the closed system of equations and boundary conditions (see details in [19]). Let

$$\vec{v} = \vec{u} + \vec{v}_p, \overline{\vec{v}_p} = 0, \quad \vec{u} = (u, v, w) = \overline{\vec{v}}, \tag{4}$$

$$p = \bar{p} + P_p, \quad \theta = T + T_p, \tag{5}$$

where the over bar denotes the average over the period of vibrations.

As shown in [19], for the description of convective flows in the liquid with free surface in the presence of high-frequency vibrations the oscillating deformations of free surface have to be taken into account. In this case, the usual Boussinesq approximation is

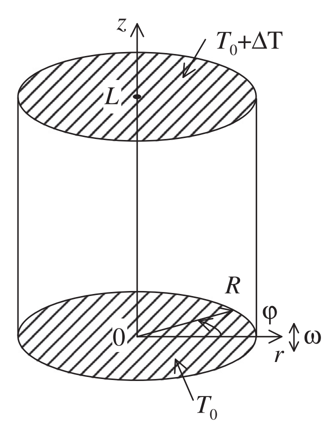

Fig. 1. geometry of the problem.