International Journal of Heat and Mass Transfer 69 (2014) 237–246

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

# New thermal effectiveness data and formulae for some cross-flow arrangements of practical interest

Luben Cabezas-Gómez [a,*], José Maria Saiz-Jabardo [b], Hélio Aparecido Navarro [c], Paulo Eduardo Lopes Barbieri [d]

[a] Department of Thermal and Fluid Sciences, Universidade Federal de São João del-Rei, UFSJ, Campus Santo Antônio, Praça Frei Orlando, 170, Centro, CEP 36307-352 São João del-Rei, MG, Brazil
[b] Escuela Politécnica Superior, Universidad de la Coruña, Mendizábal s/n Esteiro, 15403 Ferrol, Coruña, Spain
[c] Mechanical Engineering Department, Escola de Engenharia de São Carlos, Universidade de São Paulo, Av. Trabalhador São-carlense, 400, Centro, CEP 13566-590, São Carlos, SP, Brazil
[d] Centro Federal de Educação de Minas Gerais, Departamento Acadêmico de Engenharia Mecânica, Av. Amazonas, 7675, Laboratório de Refrigeração, Nova Gameleira, CEP 30510-000 Belo Horizonte, MG, Brazil

ARTICLE INFO

*Article history:*
Received 8 May 2013
Received in revised form 3 September 2013
Accepted 9 October 2013
Available online 5 November 2013

*Keywords:*
Cross-flow heat exchangers
*Z*-shape flow arrangements
Multipass parallel–counter–cross-flow arrangements
Thermal effectiveness
Number of transfer units

ABSTRACT

This study provides new effectiveness data and closed form relations for some recently proposed cross-flow heat exchanger flow arrangements of industrial and research interest. The study also covers the so called *Z*-shape flow arrangement of widespread use in the refrigeration and automobile industries. The closed form relations as well as the effectiveness data are presented in the standard effectiveness – number of transfer units ($\varepsilon$-*NTU*) format. Slight deviations have been observed of the arithmetic mean effectiveness results, proposed by Pongsoi et al. (2011, 2012) [3,4], with respect to those from both closed form relations and data from the HETE program (Navarro and Cabezas-Gómez (2005) [9], Cabezas-Gómez et al. (2007) [10]) for several flow arrangements, with deviations increasing with *NTU* and $C^*$. Data provided in the present paper, along with the closed form expressions, could be useful in enhancing the external heat transfer coefficient precision based on the procedure of determining the *NTU* value from the experimentally obtained thermal effectiveness of the heat exchanger.



## 1. Introduction

The experimental study and analysis of the thermal performance of heat transfer surfaces used in heat exchangers is one of the most important objectives of researchers and engineers in the heat transfer field. A significant number of studies have been dedicated to the experimental determination of the heat transfer characteristics of different surfaces. The books by Kays and London [1] and Webb and Kim [2] report some of the results of these investigations.

This paper aims at the development of closed form expressions for computing the thermal effectiveness of some cross-flow arrangements that have been recently investigated by Pongsoi et al. [3] and Pongsoi et al. [4–6]. These flow arrangements are typical of industrial applications, where sensible heating is a commonplace. One of such applications is waste heat recovering [3]. The so called *Z*-shape, one of the flow arrangements considered in the present paper, is extensively used in the refrigeration/air conditioning and the automobile industries.

Pongsoi et al. [3–6] investigated mixed flow arrangements known as multi-pass parallel and counter cross-flow configurations. Those configurations, except the *Z*-shape one, present a combination of parallel–cross-flow and counter–cross-flow arrangements in sequence, and could be considered as new configurations. In their investigation, Pongsoi et al. [3–6], due to the unavailability of precise thermal effectiveness correlations for the flow arrangements under consideration, suggested the effectiveness evaluation through the mean arithmetic of the thermal effectiveness of a counter–cross-flow, $\varepsilon_{c,cf}$, and a parallel–cross-flow, $\varepsilon_{p,cf}$, configurations. This fairly new approach allowed the determination of both the Number of Transfer Units, *NTU*, from the experimentally measured thermal effectiveness, and, as a consequence, the external convective heat transfer coefficient related to the investigated heat transfer surfaces.

* Corresponding author at: Department of Thermal and Fluid Sciences, Universidade Federal de São João del-Rei, UFSJ, Campus Santo Antônio, Praça Frei Orlando, 170, Centro, CEP 36307-352 São João del-Rei, MG, Brazil. Tel.: +55 31 91940649.

*E-mail addresses:* cab35ezas@yahoo.com.br, lcg@ufsj.edu.br (L. Cabezas-Gómez), jose.saiz.jabardo@udc.es (J.M. Saiz-Jabardo), han@sc.usp.br (H.A. Navarro), barbieri@des.cefetmg.br (P.E.L. Barbieri).

0017-9310/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijheatmasstransfer.2013.10.022

**Nomenclature**

| | |
|---|---|
| $A$ | exchanger outer total heat transfer area |
| $C$ | heat capacity rate |
| $C^*$ | heat capacity rate ratio, $C_{min}/C_{max}$, dimensionless |
| $N_{TL}$ | number of tube lines |
| $N_r$ | number of tube rows |
| $NTU$ | number of transfer units, $UA/C_{min}$ |
| $P$ | heat exchanger effectiveness, related to $\varepsilon$ |
| $q$ | heat transfer rate |
| $U$ | Overall heat transfer coefficient |
| $UA$ | heat exchanger conductance |

*Greek symbols*

| | |
|---|---|
| $\delta$ | relative error |
| $\varepsilon$ | conventional heat exchanger thermal effectiveness, $q/q_{max}$ |
| $\Delta$ | absolute error |

*Subscripts*

| | |
|---|---|
| *air* | external unmixed fluid, air |
| *c* | counter |
| *cf* | cross-flow |
| *m* | mean arithmetic value |
| *max* | maximum value |
| *min* | minimum value |
| *p* | parallel |
| *th* | theoretical or closed form |
| *TL* | tube lines |

As previously mentioned, in the present study, closed form ($\varepsilon$-$NTU$) relations are developed for the thermal effectiveness of the aforementioned flow arrangements. These theoretical correlations have been developed from the procedure described by Domingos [7], and applied by Shah and Pignotti [8] for similar flow arrangements (see, for example, case 5 of [8]). To the authors' best knowledge, the proposed closed form expressions have not been previously published in the open literature. The main advantage of closed form expressions is that they allow the determination of the thermal effectiveness of the particular flow configuration preventing the use of either computational or approximate approaches.

The proposed closed form relations have been evaluated through the results numerically obtained from the HETE program, Navarro and Cabezas-Gómez [9] and Cabezas-Gómez et al. [10], whose results accuracy has been demonstrated in several previous studies [9–12]. Thermal effectiveness deviations have been found between the results from the procedure suggested by Pongsoi et al. [3–6] and those from either the closed from correlations or HETE program. Though differences were generally low, under certain conditions and flow arrangements, *Z*-shape flow, for example, they can attain non negligible values. Thus, effectiveness values obtained from either the closed form relations or the HETE program could be very useful for experimental data reduction of new heat transfer surfaces.

## 2. Mean arithmetic thermal effectiveness computation

Fig. 1 displays the two-dimensional scheme of the flow arrangements investigated by Pongsoi et al. [3–6]. These configurations were experimentally studied by Pongsoi et al. [3,4] aiming at the determination of the effect of fin pitches and material and the number of tube rows on the air side thermal performance for crimped spiral fin-and-tube heat exchangers. According to these authors, there are few investigations of this kind of heat exchangers, which find industrial applications as, for example, in waste heat recovering. The studied configurations also are characterized by a mixed internal flow circuitry known as multi-pass parallel and counter cross-flow configurations [3,4]. In fact, these are new configurations, except the first one, Fig. 1(a), and consist of a combination of parallel-cross-flow and counter–cross-flow arrangements in sequence. The arrangement of Fig. 1(a) could be considered as *Z*-shape flow one.

Due to the unavailability of theoretical relations for calculating thermal effectiveness of the configurations of Fig. 1, Pongsoi et al. [3–6] suggested to evaluate them as the mean arithmetic of the counter–cross-flow, $\varepsilon_{c,cf}$, and a parallel–cross-flow, $\varepsilon_{p,cf}$, heat exchangers thermal effectiveness, that is,

$$\varepsilon_m = \frac{\varepsilon_{p,cf} + \varepsilon_{c,cf}}{2} \tag{1}$$

Note that the average thermal effectiveness is designated as $\varepsilon_m$ whereas both $\varepsilon_{p,cf}$ and $\varepsilon_{c,cf}$ should be determined for the corresponding number of tube rows of the particular flow arrangement. The tube rows stand for the columns of tubes with respect to the incoming external unmixed fluid. Several theoretical expressions from Cabezas-Gómez et al. [10] and used by Pongsoi et al. [3–6] in their procedure are shown in Table 1. Thus, in order to compute the thermal effectiveness for some flow arrangements, Eq. (1) could be used along with the corresponding equation of Table 1. According to this procedure, the thermal effectiveness of the configuration of Fig. 1(a) is determined from Eq. (1) along with Eqs. (T1.1) and (T1.5).

It is worth mentioning at this point that, for more than four rows, Pongsoi et al. [3–6] suggest that $\varepsilon_{p,cf}$, and $\varepsilon_{c,cf}$ must be determined from Eqs. (T1.4) and (T1.8), which correspond to the pure parallel and counter-flow arrangements, respectively. However, Cabezas-Gómez et al. [10] have shown that even for a number of tube rows higher than 10 the use of either the pure parallel or pure counter-flow relations could lead to non-desired effectiveness errors for high values of $NTU$ and $C^*$. These errors could be avoided if effectiveness data were obtained either from straight application of the HETE program [10] or from the closed form expressions, to be introduced further on in this paper, with support from the HETE program to determine the effectiveness for five rows parallel and counter–cross flow arrangements.

## 3. Closed form relations for computing the thermal effectiveness

Closed form expressions for the heat exchanger thermal effectiveness are developed in this section for the flow arrangements considered in the present study. To the best knowledge of the authors, these expressions have not been published in the open literature yet. In addition, they could be useful not only in thermal systems analysis but also in thermo-hydraulic evaluations of heat exchangers with similar flow arrangements. Finally these correlations, designated herein as theoretical, will be used to evaluate the precision of the thermal effectiveness obtained from both the mean arithmetic expression, Eq. (1), and the numerical one from the HETE program.

Figs. 1 and 2 present the set of flow arrangements considered in the present study, with those of Fig. 2 being for two tube lines per