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Temperature evolution of two parallel composite cylinders with contact resistances and application to thermal dual-probes

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ABSTRACT

Thermal dual-probes are frequently used to measure the heat capacity and diffusivity of soil samples. These probes mostly consist of two thin parallel needles, one serving as a heater and the other as a temperature sensor. Originally a simple infinite line source model was applied to describe the temperature evolution around the heated needle and to evaluate dual-probe measurements. Quite recently an analytical approach by Knight et al. (2012) for the infinitely long dual-probe made it possible to take the finite radius and heat capacity of the probes into account. In the present article this theory is extended to allow for surface resistances at the boundaries between the probes and the sample and, in addition, facilitating a subdivision of the probes into cylindrical cores and surrounding tubular sheaths, which is a rough representation of the needles' inner structure. In comparison with computer simulations by finite element solvers, the extended semi-analytical approach enables us to determine the influence of contact resistances with shorter computation time, providing an efficient method for the evaluation of dual-probe measurements.

1. Introduction

Since the end of the last century dual-probes have been extensively used to measure thermal properties of soils in the laboratory and field. They consist of a needle serving as a heater and a parallel needle sensing the temperature at a certain distance from the heater. Dual probes turned out to be a fast and accurate means of determining heat capacity [8]. This was an advantage over single cylinder probes which were known to provide accurate measurement of thermal conductivity but have a low accuracy for heat capacity [21]. The first dual-probe measurements were based on instantaneous heat pulses [8], i.e. using a heat impulse which, in practice, can be treated as a δ-function. Later it was recognized that the use of finite length heat pulses and data evaluation on the basis of infinite line heat pulse formulas performed better [6], giving accurate volumetric heat capacity as well as diffusivity (and thereby thermal conductivity). However, this evaluation technique is prone to a variety of errors as discussed in [10,11] and recently in [15].

The errors arising from the finite length of the probes are small when the distance between the probes is much smaller than their length. The influence of the probe radius is negligible if it is much smaller than the distance between the probes. These prerequisites can be fulfilled in practice with long thin needles. However, the remaining biases from the probe heat capacity (which may be much higher than that of the sample) and from an eventual surface (contact) resistance at the probe/sample interface are more difficult to cope with. It was only recently that the finite heat capacity and size of dual-probes were taken into account in a rigorous way by Knight et al. [12]. Thermal resistances at the probe/sample surfaces were not allowed for in their analysis. However, there is evidence that surface resistances are responsible for an underestimation of the thermal conductivity [20] and for part of the overestimation of the heat capacity [16] as measured by dual-probes. Also, single probe measurements may be corrupted by high surface resistances, whereby very long observation times are needed to determine the thermal conductivity of the sample accurately [19]. A similar assessment for the dual-probe heat pulse method was made by [18], showing that the evaluation of late-time data (temperature in the sensing probe after the signal has dropped to about 90% of its maximum value) reduces the overestimation of specific heat. The downside is that the finite probe lengths, finite sample container width and axial heat loss may play a significant role with very long observation times [3–5]. On the other hand, early-time data, which can be used in single probe measurements for determining the surface conductance [16], may crucially depend on the inner probe structure.

To cope with the above mentioned difficulties, one has to develop a formalism which allows the use of small, intermediate and large time data (only excluding very late times where the above mentioned effects are significant). For that purpose we merge the approaches of Knight et al. [12] and Macher et al. [19] to obtain

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a semi-analytic description of the temperature in two cylinders, each of which is subdivided into a core and a mantle (sheath), with surface resistances allowed at all boundaries. Details of the geometry are given in Section 2. The pertinent governing equations are solved after Laplace transformation (Sections 3–5). The time-dependent solutions are calculated by numerical inverse Laplace transformation, which is explained in Section 6, proving that the integration path can be modified in a suitable way to ensure numerical convergence and fast computation time. In Section 7 an application of the theory to a typical dual-probe illustrates the importance of surface resistances.

2. Geometry

We investigate two infinite cylinders A and B which are parallel to the z-axis, with their axes at a distance L from each other. The cylinder axes cross the x-axis at $-L/2$ and $L/2$, respectively. Each cylinder is composed of two parts, a core and an adjacent tubular sheath. A cross-section of the geometry is shown in Fig. 1. Surface resistances are allowed between the cylinder cores and their tubular sheaths, as well as between the sheaths and the sample medium. Analysis of a single cylinder with this structure was performed in [19]. Here we consider two cylinders of this form which need not have the same specifications. In detail, the relevant parameters are:

$a_1^A,\ a_1^B$	core radii
$a_2^A,\ a_2^B$	outer sheath radii
$S_1^A,\ S_1^B$	core heat capacities per length
$S_2^A,\ S_2^B$	sheath heat capacities per length
$H_1^A,\ H_1^B$	surface conductances core/sheath
$H_2^A,\ H_2^B$	surface conductances sheath/sample
$Q_1^A,\ Q_1^B$	heating power supplied to cores per length
$Q_2^A,\ Q_2^B$	heating power supplied to sheaths per length
$r^A,\ r^B$	distances from cylinder axes
$\phi^A,\ \phi^B$	azimuth with cylinder axes as origin
$T_1^A(t),\ T_1^B(t)$	temperature evolution in cores
$T_2^A(t), T_2^B(t)$	temperature evolution in sheaths

Subscripts 1 and 2 refer to core and sheath, respectively, superscripts A and B indicate the corresponding probe. For instance, the heat capacity S_1^A denotes the heat stored per unit length in the core of cylinder A when the temperature is increased by 1 K. The surface conductances define the power transfer via the respective boundary per unit area and temperature difference. These boundaries are the interfaces between the cores and the respective sheaths, and the sheaths and the surrounding medium.

We assume that the core and sheath are perfect thermal conductors, i.e. $\lambda_1^A = \lambda_1^B = \lambda_2^A = \lambda_2^B = \infty$. This assumption is applicable in many dual probe measurements, as was demonstrated in [12] for the dual probe heat pulse method. The authors investigated dual probes with epoxy cores ($\lambda_1^A = \lambda_1^B = 1.04$ W/m K) and stainless steel sheaths ($\lambda_2^A = \lambda_2^B = 14.9$ W/m K). Their numerical simulations showed that assuming infinite conductivity (actually 10^5 W/m K) of cores and sheaths (for a dual-probe measurement as treated in Section 7) causes a maximum temperature bias in the sensing probe B of 0.016 K for dry sand, 0.012 K for wet sand and 0.004 K for water samples, which was about 1% of the respective maximum temperature increase. For the most part of the temperature evolution the bias was much smaller and below the measurement accuracy. These studies did not take surface resistances into account, which decrease the heat flow between core, sheath and sample, and so facilitate the temperature balance within the probe parts. We can therefore expect that surface resistances reduce the influence of the finite probe conductivity even more. Under such circumstances the infinite conductivity assumption is therefore well justified.

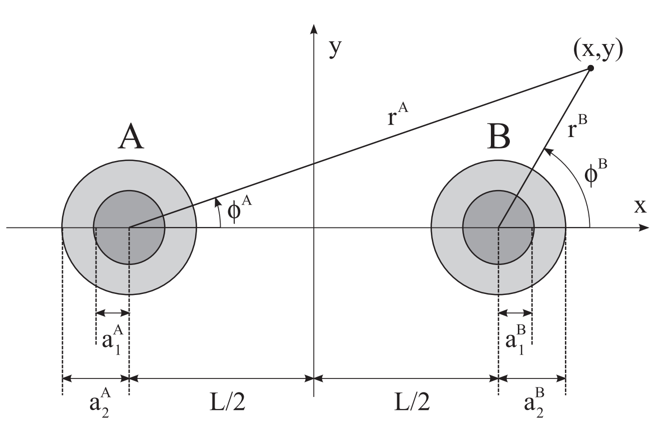

Fig. 1. Cross-section of dual-probe cylinders A and B.

Two types of cylinder coordinates (r^A, ϕ^A) and (r^B, ϕ^B) are introduced, with the respective cylinder axis as reference (see Fig. 1 for a graphical definition).

$$r^A = \sqrt{(x + L/2)^2 + y^2} \tag{1}$$

$$r^B = \sqrt{(x - L/2)^2 + y^2} \tag{2}$$

$$\tan \phi^A = y/(x + L/2) \tag{3}$$

$$\tan \phi^B = y/(x - L/2) \tag{4}$$

Although the arrangement is symmetric with regard to the yz-plane and the xz-plane, the temperature evolution will only adopt the latter symmetry. The reason is that cylinder A is used as a heater and cylinder B as a sensing probe, with heat flowing from the left to the right hemisphere.

3. Mathematical statement of the problem

The cylinders are placed in an external medium of infinite extent, which is specified by its thermal conductivity λ and diffusivity κ. The medium is initially in thermal equilibrium. We further assume that its initial temperature T_0 is zero, which amounts to saying that all temperature values encountered in the following are offsets from T_0. This procedure is valid because the differential equation and the boundary conditions are invariant under a shift of all temperatures by the same value.

Let the initial temperatures (at $t = 0$) of the cylinder parts be denoted by $T_{n0}^A = T_n^A(0)$ and $T_{n0}^B = T_n^B(0) (n \in \{1, 2\})$. Probe A is heated, supplying a power Q_1^A to the core and Q_2^A to the sheath per cylinder length. Arbitrary initial temperatures of the core and sheath are allowed. Probe B serves only as a temperature sensor (no heating), initially at the same temperature as the external medium: $Q_1^B = Q_2^B = 0,\ T_{10}^B = T_{20}^B = T_0 = 0$.

Since we consider probes of infinite length, the z coordinate disappears from the problem, and the temperature $T(x, y, t)$ in the medium depends only on the coordinates x and y, and on time t. The temperature $T(x, y, t)$ around the two probes satisfies the heat diffusion equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \frac{1}{\kappa}\frac{\partial T}{\partial t} = 0 \tag{5}$$

If there were only one probe, say probe A, the temperature $T^\bullet$ around this heated element would evolve in a radially symmetric fashion, and the differential equation could be simplified by introducing the cylindrical coordinates (r^A, ϕ^A), giving

$$\frac{\partial^2 T^\bullet}{\partial (r^A)^2} + \frac{1}{r^A}\frac{\partial T^\bullet}{\partial r^A} - \frac{1}{\kappa}\frac{\partial T^\bullet}{\partial t} = 0 \tag{6}$$