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Technical Note

New configuration factor between a circle and a point-plane at random positions

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ABSTRACT

Following complex mathematical analysis, the authors have found exact expressions for the elusive configuration factor between a circle and the three coordinate axes. This new factor will bring considerable independence and versatility to radiative transfer analysis. However, it is still necessary for many engineering and architectural applications to find the value of the said factor for points lying on a plane at an arbitrary position i.e. askew to the former reference directions. By virtue of the radiation vector theory, the calculation is performed whether the angles that the plane forms with the previous axes are known. If the receiving plane cuts the disk in two halves a new original factor is also presented.

1. Introduction

The finding of configuration factors between an emitting circle and a point that lies in a plane perpendicular or parallel to the source has been a difficult task for both lighting designers and scientists. Exact expressions derived from double integration were not accessible. Likewise it happened to those calculations for a differential element in a reference system other than the one of the disk.

These factors open interesting possibilities about the radiative performance of diverse emitters and its interaction with surfaces randomly placed around them. For instance, the authors deem that some figures which radiate in a three-dimensional manner could be decomposed, with sufficient accuracy, into disks of different size, comparable to circular finite elements. Such procedure would provide versatile approaches to radiative transfer simulations.

Extensive and remarkable researches have been performed on the matter, from the 20th century to our days, by several authors. Nevertheless, as a complete understanding of the radiative behavior of the circles was not achieved, classical heat transfer handbooks and scientific papers have not properly solved the issue thus far and only could reach partial solutions with limited range of applicability; for instance, according to Naraghi [1], Naraghi and Chung [2], the configuration factor between an emitting disk and a differential element rotated an arbitrary angle Θ but only in the *x–z* plane is:

For $\tan^{-1}\left(\frac{H}{A-1}\right) \le \theta \leqslant \tan^{-1}\left(\frac{H}{A+1}\right)$:

$$F_{d1-2} = \frac{1}{2}\left[\left(\frac{1-A^2-H^2}{Z}+1\right)\cos\theta + \frac{H}{A}\left(1-\frac{X}{Z}\right)\sin\theta\right]$$

For $0 \le \theta \le \pi$ or $\tan^{-1}\left(\frac{\mathrm{H}}{\mathrm{A}+1}\right) \le \theta \le \tan^{-1}\left(\frac{H}{A-1}\right)$:

$$F_{d1-2} = \frac{(1-A^2-H^2)\cos\theta - \frac{XH\sin\theta}{A}}{\pi Z}\tan^{-1}\left[\left(\frac{X+2A}{X-2A}\right)^{1/2}\cot\left(\frac{\cos^{-1}Y}{2}\right)\right] + \frac{1}{2\pi}\left(\cos\theta + \frac{H}{A}\sin\theta\right)(\pi - \cos^{-1}Y) + \frac{1}{\pi}\tan^{-1}\left[\frac{\sin\theta}{H}(1-Y^2)^{1/2}\right] \quad (1)$$

Being the factors involved:

$$A = A = a/r \quad H = h/r \quad X = 1 + A^2 + H^2$$
$$Y = H \cdot cot\theta - A \quad Z = \sqrt{X^2 - 4A^2} \quad (2)$$

This equation solely provides the configuration factor for a differential element tilted in the *x–z* plane and aligned with respect to the center of the emitting circle.

In a similar fashion, the configuration factor between an emitting circle and a differential element tilted an arbitrary angle in a fixed plane of symmetry is, according to Hollands [3]

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$$F_{d1-2} = \frac{1}{2}\left[(\cos\theta_k - H\cos\theta_j) + \frac{HZ\cos\theta_j}{P} - \frac{(1+H^2-R^2)}{P}\cos\theta_k\right] \quad (3)$$

As in [1], the formula is heavily limited by the fact that the y–z plane that comprises the differential element has to contain the center of the disk. Complete references about these factors, with detailed explanation and graphical references for all the proposed expressions can be found at [4].

2. Calculation process

According to previous researches by the authors [5–7], the configuration factor between an emitting circle and a point located in a perpendicular or parallel plane for the principal directions of space has been deducted via analytical exact expressions, completing the results already obtained by former authors [8]. Their three components are:

$$F_y = \frac{1}{2}\left(1 - \frac{x^2+y^2+z^2-r^2}{\sqrt{(r^2+y^2+z^2)^2 - 4*r^2*(x^2+z^2)}}\right) \quad (4)$$

In the y direction.

$$F_z = \frac{yz}{2(x^2+z^2)}\left[\frac{r^2+x^2+y^2+z^2}{\sqrt{(r^2+x^2+y^2+z^2)^2 - 4*r^2*(x^2+z^2)}} - 1\right] \quad (5)$$

In the z direction.

$$F_x = \frac{yx}{2(x^2+z^2)}\left[\frac{r^2+x^2+y^2+z^2}{\sqrt{(r^2+x^2+y^2+z^2)^2 - 4*r^2*(x^2+z^2)}} - 1\right] \quad (6)$$

For the x direction

These two latter factors, unavailable in the literature, have been derived via symbolic calculus by the authors [5] and are presented in simple Cartesian parameters for the first time.

The radiation vector theory states that once the components of the vector are known for three coordinate planes, the said vector can be found for any other direction whose angles are defined in the problem.

Given the expressions [4–6], we are able of constructing the radiation vector at any point using vector addition.

$$\overleftarrow{F} = \overleftarrow{F_x} + \overleftarrow{F_y} + \overleftarrow{F_z} \quad (7)$$

Before constructing the vector, some simplifications could be made. Firstly, we would rename its components:

By making

$$P = r^2 + x^2 + y^2 + z^2 \quad (8)$$

and

$$Q = \sqrt{(r^2+x^2+y^2+z^2)^2 - 4r^2\cdot(x^2+z^2)} \quad (9)$$

The three factors are simplified to,

$$F_x = \frac{yx}{2(x^2+z^2)}\left[\frac{P}{Q} - 1\right]$$

$$F_y = \frac{1}{2}\left(1 - \frac{P-2r^2}{Q}\right)$$

$$F_z = \frac{yz}{2(x^2+z^2)}\left[\frac{P}{Q} - 1\right] \quad (10)$$

Thus, the configuration factor that we are seeking is,

$$F = \frac{yx}{2(x^2+z^2)}\left[\frac{P}{Q} - 1\right] + \frac{1}{2}\left(1 - \frac{P-2r^2}{Q}\right) + \frac{yz}{2(x^2+z^2)}\left[\frac{P}{Q} - 1\right] \quad (11)$$

Let us develop each one of the former components to find the modulus. For the first one F_x we have:

$$[F_x]^2 = \frac{y^2x^2(P^2+Q^2-2PQ)}{2Q^2(x^2+z^2)^2} \quad (12)$$

For the second one, that is, F_y:

$$[F_y]^2 = \left[\frac{1}{2}\left(1 - \frac{P-2r^2}{Q}\right)\right]^2 = \left[\frac{1}{2} - \frac{P-2r^2}{2Q}\right]^2 = \frac{1}{4} + \frac{P^2+4r^2-4Pr^2}{4Q^2} - \frac{2P-4r^2}{4Q} = \frac{Q^2+P^2+4r^2-4Pr^2-2PQ-4Qr^2}{4Q^2} = \frac{(Q-P-2r^2)^2}{4Q^2} \quad (13)$$

For F_z, the operation is similar to F_x, only changing z for x, and that yields:

$$[F_z]^2 = \frac{y^2z^2(P^2+Q^2-2PQ)}{2Q^2(x^2+z^2)^2} \quad (14)$$

And now we only need to assemble the former terms into one single expression:

$$|\overleftarrow{F}| = \sqrt{(F_x)^2+(F_y)^2+(F_z)^2} = \sqrt{\frac{y^2x^2(P^2+Q^2-2PQ)}{2Q^2(x^2+z^2)^2} + \frac{(Q-P-2r^2)^2}{4Q^2} + \frac{y^2z^2(P^2+Q^2-2PQ)}{2Q^2(x^2+z^2)^2}} \quad (15)$$

Rearranging and operating with [15] we can reach a more simplified equation:

$$|\overleftarrow{F}| = \sqrt{\frac{y^2x^2(P^2+Q^2-2PQ)+y^2z^2(P^2+Q^2-2PQ)+(x^2+z^2)^2(Q-P-2r^2)^2}{4Q^2(x^2+z^2)^2}} = \sqrt{\frac{y^2(P^2+Q^2-2PQ)(x^2+z^2)+(x^2+z^2)^2(Q-P-2r^2)^2}{4Q^2(x^2+z^2)^2}} \quad (16)$$

$$\frac{\sqrt{y^2(x^2+z^2)\left((r^2+x^2+y^2+z^2)^2 + (r^2+x^2+y^2+z^2)^2 - 4r^2\cdot(x^2+z^2) - 2(r^2+x^2+y^2+z^2)\sqrt{(r^2+x^2+y^2+z^2)^2 - 4r^2\cdot(x^2+z^2)}\right) + (x^2+z^2)^2\left(\sqrt{(r^2+x^2+y^2+z^2)^2 - 4r^2\cdot(x^2+z^2)} - 3r^2 - x^2 - y^2 - z^2\right)^2}}{2(x^2+z^2)\left(\sqrt{(r^2+x^2+y^2+z^2)^2 - 4r^2\cdot(x^2+z^2)}\right)} \quad (17)$$