International Journal of Heat and Mass Transfer 63 (2013) 313–322

Contents lists available at SciVerse ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Numerical solution of a thermal instability problem in a rotating nanofluid layer

Dhananjay Yadav [a,*], R. Bhargava [a], G.S. Agrawal [b]

[a] Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee, India
[b] Department of Computer Science, Manglayatan University, Aligarh, India

ARTICLE INFO

Article history:
Received 18 August 2012
Received in revised form 15 December 2012
Accepted 4 April 2013
Available online 30 April 2013

Keywords:
Nanofluid
Brownian motion
Thermal instability
Rotation
Galerkin method
Critical Rayleigh number
Thermophoresis

ABSTRACT

Thermal instability of a nanofluid layer heated from below in the presence of rotation is investigated. The lower boundary of the nanofluid layer is considered to be rigid, while the upper boundary is assumed to be either rigid or free. The effects of Brownian motion and thermophoresis have been included in the model of nanofluid. The 6-term Galerkin method is used to obtain the eigenvalue equation, which is then solved numerically. The effects of rotation and other physical parameters on the onset of convection are analyzed and compared for two types velocity boundary conditions considered. Besides, some known results available in the literature are compared with those obtained from the present study and good agreement is found.

1. Introduction

Investigation of the effects of rotation and boundary on the onset of convection was started several decades ago [1–9]. Recently, there has been significant interest in nanofluids. This interest is generated by a variety of applications in industrial, commercial, residential and transportation sectors [10]. The term "nanofluid" is a usual heat transfer fluids with suspended ultra fine particles of nanometer size. The term was coined by Choi [11].

Initiating the analysis of convective instability and heat transfer characteristics of nanofluids, Kim et al. [12] have shown that as the density and heat capacity of nanoparticles increases and the thermal conductivity and the shape factor of nanoparticles decrease, the convective motion in a nanofluid set is easily. Alloui et al. [13] studied the natural convection of nanofluids in a shallow cavity heated from below. They observed that the presence of nanoparticles in a fluid is found to reduce the strength of flow field, this behavior being more pronounced at low Rayleigh number. Also, the temperatures on the solid boundaries are reduced (enhanced) by the presence of the nanoparticles when the strength of convection is high (low).

Tzou [14,15] studied thermal instability problems of nanofluid with rigid-free [14] and free-free [15] boundaries using method of eigenfunction expansions. Dhananjay et al. [16] studied Rayleigh–Bénard convection in nanofluid for free-free boundaries using Galerkin method and they have also discussed the case of overstability that has not been studied by previous author Tzou [14,15]. The onset of convection in a horizontal nanofluid layer of finite depth was studied by Nield and Kuznetsov [17]. It is found that the critical Rayleigh number can be reduced or increased by a substantial amount, depending on whether the basic nanoparticle distribution is top-heavy or bottom-heavy, by the presence of nanoparticles. Nield and Kuznetsov [18] have studied the onset of double-diffusive convection in a nanofluid layer. On using one-term Galerkin approximation, they obtained the stability boundaries for both non-oscillatory and oscillatory cases. The analytical results for oscillatory stability were limited to the case of large Prandtl number and large nanoparticle Lewis number. Thermal instability of rotating nanofluid layer for free-free boundaries has been studied by Yadav et al. [19]. They observed that for non oscillatory convection, rotation and the difference on the temperature have stabilizing effects while the combine behaviours of Brownian motion and thermoporesis of nanoparticles creates destabilizing effect. Yadav et al. [20] have also examined the effects of boundary and internal heat source on the onset of Darcy–Brinkman convection in a porous layer saturated by nanofluid and obtained that the presence of constant internal heating makes both basic temperature distribution and basic volumetric fraction of nanoparticles distribution to deviate from linear to nonlinear and the critical Rayleigh number decreases with as increase in the constant

* Corresponding author. Tel.: +91 9286685908.
E-mail addresses: dhananjayadav@gmail.com, dyadadma@iitr.ernet.in (D. Yadav), rbharfma@iitr.ernet.in (R. Bhargava), gsa45fma@gmail.com (G.S. Agrawal).

0017-9310/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijheatmasstransfer.2013.04.003

internal heat source strength. On the onset of thermal convection in rotating nanofluid layer saturating a Darcy–Brinkman porous medium for free-free boundaries was studied by Chand and Rana [21]. Thermal instability in a nanofluid layer with vertical magnetic field for free-free, rigid-rigid and rigid-free boundaries was studied by Yadav et al. [22]. Agarwal et al. [23] and Bhadauria and Agarwal [24] studied the same problem of thermal instability in a rotating porous layer saturated by a nanofluid for top-heavy and bottom-heavy suspension for the Darcy model and for the Brinkman model for bottom-heavy suspension, respectively. Nield and Kuznetsov [25] investigated the natural convection for flow in a porous medium saturated by nanofluid using the Darcy model. The extension to the Brinkman model was made by Kuznetsov and Nield [26]. Kuznetsov and Nield [27,28] also studied local thermal non-equilibrium and flow past vertical plate for nanofluids. The onset of double-diffusive nanofluid convection in a layer of porous medium was studied by Kuznetsov and Nield [29] using the Darcy model. The extension to the Brinkman model was made by Yadav et al. [30].

In the present analysis, we have studied the onset of convection in a rotating nanofluid layer numerically. Numerical results are presented for Al_2O_3–water nanofluid and Cu–water nanofluid. The effects of rotation and other physical parameters on the onset of convection have been also analyzed for water nanofluids with metallic/metallic oxide nanoparticles keeping other parameters as constant. Al_2O_3–water nanofluid and Cu–water nanofluid are the most common nanofluids and highly important because they can be used in numerous applications involving heat transfer and other applications [31,32].

The aluminum nanoparticles are covered with thin layers of aluminum oxide, owing to the high oxidation activity of pure aluminum, thus creating a larger contact surface area with water and allowing for increased decomposition of hydrogen from water during the combustion process. During this combustion process, the alumina acts as a catalyst and the aluminum nanoparticles then serve to decompose the water to yield more hydrogen. It was shown that the combustion of diesel fuel mixed with aqueous aluminum nanofluid increased the total combustion heat while decreasing the concentration of smoke and nitrous oxide in the exhaust emission from the diesel engine [33].

The behavior and heat transfer enhancement of Al_2O_3–water nanofluid, flowing inside a closed system that is destined for the cooling of microprocessors or other electronic components was experimentally studied by Nguyen et al. [34]. The experimental data supports that the inclusion of nanoparticles into distilled water produces a significant increase of the cooling convective heat transfer coefficient. At a given particle concentration of 6.8%, the heat transfer coefficient increased as much as 40% compared to the base fluid of water.

In another experiment, You et al. [35] measured the enhancement of the critical heat flux in pool boiling from a flat square heater immersed in Al_2O_3–water nanofluid. The test results showed that the enhancement of CHF was drastic when nanofluid was used as a cooling liquid instead of pure water. It was concluded that the increase in CHF levels present the possibility of raising chip power in electronic components or simplifying cooling requirements for space applications.

The heat transfer coefficient of a Cu–water nanofluid through a circular tube with a constant wall heat flux boundary condition was experimentally investigated by Li and Xuan [36]. They obtained the variation from 1.05 to 1.14 for the ratio of the nanofluid Nusselt number to that of pure water by increasing the volume concentration of nanoparticle from 0.5% to 1.2%, respectively.

It is a known fact that the types of bounding surfaces play a significant role on the onset of convection in a nanofluid layer and several studies have been under completed in the past without rotation [17,18,20,22,26,30]. However, due attention has not been given to such a study using a rotation and there is sufficient scope for further study. Such a study finds its relevance in innumerable practical application in modern science and engineering, including rotating machineries like nuclear reactors, petroleum industry, biochemical and geophysical problems. The aim of the present paper is to study the effects of rotation on the onset of convection in a nanofluid layer for realistic boundary conditions, namely (i) both boundaries rigid, and (ii) lower rigid and upper free boundaries by specifying constant but different nanoparticles volumetric fraction and temperatures at the boundaries. Unlike the free-boundaries case, the solution for the eigenvalue problem in exact form is not possible for these two sets of boundary conditions and hence the resulting eigenvalue problem is solved numerically by employing a higher-order Galerkin method.

2. Formulation of the problem

Consider an infinite horizontal layer of incompressible nanofluid which is kept rotating about vertical axis at a constant angular velocity $\mathbf{\Omega}^* = (0, 0, \Omega^*)$, and heated from below. Let us consider the Cartesian coordinate system x, y, z in which z axis is taken at right angle to the boundaries. The nanofluid is confined between two parallel plates $z^* = 0$ and $z^* = L$, where temperature and volumetric fraction of nanoparticle are kept constants: $T^* = T_0^*,\ \phi^* = \phi_0^*$ at $z^* = 0$ and $T^* = T_1^*(< T_0^*),\ \phi^* = \phi_1^*$ at $z^* = L$, as shown in the Fig. 1. Asterisks are used to distinguish the dimensional variables from the non-dimensional variables (without asterisks). It is assumed that the Oberbeck–Boussinesq approximation [37] is valid. The conservation equations as formulated by Buongiorno [38] are now extended as follows.

The continuity equation for the incompressible nanofluid is

$$\nabla^* \cdot \vec{\mathbf{v}}^* = 0, \tag{1}$$

If one introduces a buoyancy force and Coriolis force, and adopts the Boussinesq approximation, then the momentum equation can be written as:

$$\rho_0 \left[\frac{\partial}{\partial t^*} + (\vec{\mathbf{v}}^* \cdot \nabla^*) \right] \vec{\mathbf{v}}^* = -\nabla^* p^* + \mu \nabla^{*^2} \vec{\mathbf{v}}^* + \rho \vec{\mathbf{g}} + 2\rho_0 \mathbf{v}^* \times \vec{\mathbf{\Omega}}^*, \tag{2}$$

where, ρ_0 is the nanofluid density at reference temperature T_0^*, p^* is the pressure, μ and ρ are the viscosity and density of nanofluid respectively. The nanofluid density (ρ) and reference density of nanofluid (ρ_0) are given as [39]:

$$\rho = \phi^* \rho_p + (1 - \phi^*) \rho_f, \tag{3}$$

$$\rho_0 = \phi^* \rho_p + (1 - \phi^*) \rho_{f0}, \tag{4}$$

ρ_p and ρ_f are the density of the nanoparticles and base fluid respectively, ρ_{f_0} are the base fluid density at reference temperature T_0^*.

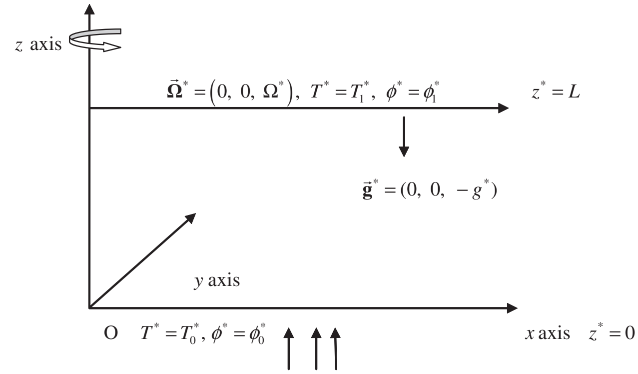

Fig. 1. Sketch of the problem geometry and coordinates.