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Study of the one dimensional and transient bioheat transfer equation: Multi-layer solution development and applications

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ABSTRACT

In this work we derive an analytical solution given by Bessel series to the transient and one-dimensional (1D) bioheat transfer equation in a multi-layer region with spatially dependent heat sources. Each region represents an independent biological tissue characterized by temperature-invariant physiological parameters and a linearly temperature dependent metabolic heat generation. Moreover, 1D Cartesian, cylindrical or spherical coordinates are used to define the geometry and temperature boundary conditions of first, second and third kinds are assumed at the inner and outer surfaces. We present two examples of clinical applications for the developed solution. In the first one, we investigate two different heat source terms to simulate the heating in a tumor and its surrounding tissue, induced during a magnetic fluid hyperthermia technique used for cancer treatment. To obtain an accurate analytical solution, we determine the error associated with the truncated Bessel series that defines the transient solution. In the second application, we explore the potential of this model to study the effect of different environmental conditions in a multi-layered human head model (brain, bone and scalp). The convective heat transfer effect of a large blood vessel located inside the brain is also investigated. The results are further compared with a numerical solution obtained by the Finite Element Method and computed with COMSOL Multiphysics v4.1©.

1. Introduction

The heat transfer in living tissues, known as bioheat transfer, is a complex phenomenon that depends on the thermodynamics of the biological system, its thermal constitutive parameters and the thermal response to external stimulus, e.g., electromagnetic or ultrasonic waves used in cancer treatments [1–4].

The study of bioheat transfer is especially relevant to the field of thermal medicine, since experimental temperature data is not extensively available. Temperature measurement techniques are mostly invasive as well as expensive and provide a limited number of measurement points. Non-invasive temperature measurement techniques, such as magnetic resonance thermal imaging, allow volumetric temperature measurements. However, they are limited due to its high cost and low thermal resolution [3,5].

Several therapeutic applications based on the knowledge of bioheat transfer involve either raising or lowering temperature from normal body temperature, namely, hyperthermia [1–3] and hypothermia [4], respectively. Hyperthermia may be defined as raising the temperature of a certain region of the body above normal for a defined period of time, usually between 30 and 90 min [1]. The most common techniques to induce hyperthermia are based on heat deposition from electromagnetic [3,5] or ultrasound sources [2], where the biological tissues convert the absorbed energy into heat causing a temperature increase. Another approach to heat generation involves injecting magnetic nanoparticles immersed in fluid into the target tissue to absorb energy at a higher rate than the surrounding tissue from an externally applied electromagnetic field [6–9]. This technique is known as magnetic fluid hyperthermia (MFH).

The efficacy of hyperthermia for cancer therapy is dependent on the delivery of well-controlled moderate heating (approximately 42 °C) to the entire tumor volume without overheating the surrounding critical normal tissues [3,10]. This technique takes advantage of the rapid neoplastic cell growth, which makes it more sensitive to an increase of temperature [11]. To optimize new hyperthermia based procedures, it is essential to develop a simplified but accurate model to estimate the temperature distribution and highlight the overall effect of the various parameters.

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Nomenclature

i	tissue layer index	ρ_i	mass density (kg m^{-3})
r^*	spatial coordinate (m)	c_i	specific heat capacity (J kg^{-1} K^{-1})
t^*	time (s)	k_i	thermal conductivity (W m^{-1} K^{-1})
T_i^*	temperature (K)	ω_{bi}	blood perfusion (s^{-1})
T_{0i}^*	initial temperature (K)	ρ_b	mass density of blood (kg m^{-3})
T_a^*	arterial blood temperature (K)	c_b	specific heat capacity of blood (J kg^{-1} K^{-1})
P_i^*	internal heat generation (W m^{-3})	Q_{mi}^*	metabolic heat generation (W m^{-3})

In 1948, Pennes was the first to propose and validate experimentally an analytical bioheat transfer model with a heat loss term due to blood perfusion [12]. Besides perfusion, Pennes' model also accounted for thermal storage, heat conduction and heat generation caused by internal and/or external sources. Other accurate bioheat transfer models have been suggested [13,14]. However, Pennes' model is the most widely used because of its simplicity and acceptable accuracy if no large thermally significant blood vessels are close to the analyzed heated region [12,13].

Solutions of the Pennes' bioheat equation were obtained in regions with Cartesian, cylindrical and spherical geometries [6–8,15–18]. Durkee et al. derived an analytical solution of the classical bioheat equation using eigenfunctions for spherical and Cartesian coordinates in the reference [15] and cylindrical coordinates in reference [16]. In both cases, a constant heat source term was used. In reference [17] Durkee et al. used Green functions to solve the classical bioheat equation for time dependent sources. Continuity boundary conditions to the temperature and heat flow were imposed at the interfaces. Moreover, Neumann and Robin boundary conditions at the inner and outer surfaces were assumed, respectively.

Bagaria and Johnson [7] used the method of separation of variables to obtain a transient and 1D solution to estimate the temperature in two concentric spherical regions. They assumed that the tumor was located in the inner region containing magnetic nanoparticles only with a polynomial distribution. On the other hand, a source term described by an exponential function was validated experimentally by Salloum et al. [9].

The purpose of this work is to derive an analytical solution to the transient and one-dimensional Pennes' bioheat equation in a multi-layer region with generic spatially dependent heat sources. Each region represents an independent biological tissue (e.g., skin, fat or muscle) characterized by temperature-invariant physiological parameters and linearly temperature dependent metabolic heat generation. Moreover, 1D Cartesian, cylindrical or spherical coordinates are used to define the geometry and continuity boundary conditions are imposed to the temperature and heat flow between adjacent layers. The inner and outer surfaces satisfy equations with adaptable parameters that allow one to define Dirichlet, Neumann and/or Robin boundary conditions.

This bioheat transfer model, which makes use of a formalism previously described by Rodrigues et al. [19], is applied to obtain the theoretical temperature profiles in the tumor bed and surrounding healthy tissue using two spatially dependent heat source terms to simulate a MFH treatment. We further explore the potential of this model to study the effect of different environmental conditions in a multi-layered human head model (brain, bone and scalp). The convective effect of a large blood vessel located inside the brain is also investigated assuming a laminar and fully thermally developed blood flow.

Furthermore, we use two approaches to validate the analytical solution. In the first one, we determine the error associated with the truncated Bessel series that defines the transient solution whereas in the second one we compare the analytical and numerical solutions. These numerical solutions are obtained using the Finite Element Method (FEM) and computed with COMSOL Multiphysics v4.1©.

2. Mathematical formulation

2.1. Pennes bioheat transfer equation

The bioheat transfer equation in a multi-layer region is given by

$$\rho_i c_i \frac{\partial T_i^*}{\partial t^*}(r^*,t^*) = k_i \nabla^2 T_i^*(r^*,t^*) + \omega_{bi}\rho_b c_b\left(T_a^* - T_i^*(r^*,t^*)\right) + Q_{mi}^*(r^*,t^*) + P_i^*(r^*) \tag{1}$$

where

$$\nabla^2 T_i^*(r^*,t^*) = \frac{1}{r^{*G}}\frac{\partial}{\partial r^*}\left(r^{*G}\frac{\partial T_i^*}{\partial r^*}(r^*,t^*)\right) \tag{2}$$

$$Q_{mi}^*(r^*,t^*) = Q_{m0i}^* + Q_{msi}^* T_i^*(r^*,t^*) \tag{3}$$

with $1 \leqslant i \leqslant n, r_{i-1}^* \leqslant r^* \leqslant r_i^* (n \in \mathbb{N})$ and $G = 0$, 1 and 2 for problems with 1D Cartesian, cylindrical and spherical symmetric geometries, respectively. Note that the mathematical method prescribed here works for a constant $(Q_{mi}^* = Q_{m0i}^*)$ or linearly temperature dependent metabolic heat generation given by (3) with a slope denoted by Q_{msi}^* [20].

The tissue temperature described by (1) is controlled by heat storage $(\rho_i c_i(\partial T_i^*/\partial t^*))$, thermal conduction $\left(k_i\nabla^2 T_i^*\right)$, dissipation of heat through blood flow $(\omega_{bi}\rho_b c_b(T_a^* - T_i^*))$ and heat generation (P_i^*), which represents the contribution from volumetric heat generation, converted from some other form of energy such as electromagnetic, ultrasonic or other modes of heating. Metabolic heat generation (Q_{mi}^*) is another type of heat input resulting from biochemical conversion of energy within tissue [11,12].

As example, a region with n layers and a spherically symmetric geometry is presented in Fig. 1. These layers correspond to biological tissues (e.g., skin, fat and muscle) and r_0^* can be equal to zero or $r_0^* > 0$ m to take into account air or liquid body regions as well as catheters like those used in transurethral prostatic microwave thermotherapy [21].

2.2. Boundary and initial conditions

Boundary conditions of first, second and third kinds to the temperature at the inner and outer surfaces are assumed (see (4) and (5)). Temperature and heat flow must satisfy continuity boundary conditions at the tissue interfaces (see (6)–(9)). An initial spatially dependent temperature is also considered (see (10)).

- Inner surface of 1st layer ($i = 1$)

$$A_{in}^* \frac{\partial T_1^*}{\partial r^*}\left(r_0^*,t^*\right) + B_{in}^* T_1^*\left(r_0^*,t^*\right) = C_{in}^* \tag{4}$$