International Journal of Heat and Mass Transfer 62 (2013) 163–173

Contents lists available at SciVerse ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

On microchannel shapes in liquid-cooled electronics applications

Burt S. Tilley

Department of Mathematical Sciences, Worcester Polytechnic Institute, 100 Institute Road, Worcester, MA 01609, USA

ARTICLE INFO

Article history:
Received 9 August 2012
Received in revised form 9 February 2013
Accepted 12 February 2013
Available online 22 March 2013

Keywords:
Lubrication theory
Homogenization
Liquid-cooled electronics

ABSTRACT

We consider the effect of modulating the microchannel shapes within a liquid-cooled device to determine how geometry affects heat transfer effects. By using the asymptotic approach of homogenization, we find a single advection–diffusion equation, which has an anisotropic thermal conductivity tensor that depends on the local channel width and laminate thickness. A novel modification of the homogenization approach is used to allow the spacing of the microchannels and laminates to vary over the macroscale, relaxing the spatial periodicity requirement of the technique. We find that the anisotropic thermal conductivity corresponds to two different classes of steady thermal solutions: a hot spot symmetric along the centerline, or two maxima which are symmetric about the centerline. The location of the hot spot depends on the energy balance between convective transport in the streamwise direction and effective conduction in the spanwise direction. Optimiziation formulations for families of channel/laminate geometries are performed to find minimum average temperatures within each of these families.

1. Introduction

As the need for more sophisticated electronic devices increases, their power needs increase as well. One limitation on the development of these devices is the requirement to remove heat energy from the system in order to maintain consistent operation. Applications where the rates of heat transport are important in the field are in radar stations and vision systems, to name only a few. Traditional heat fins which use air convection to dissipate the energy are not sufficient to remove the required energy from these devices.

A different paradigm was proposed by Tuckerman and Pease [1] to use microchannels within the electronic device to carry the energy from the chip to the environment. In the past 30 years, the need for small-scale heat exchangers has increased significantly, but there has been controversy on whether macroscale flow and thermal correlations are appropriate for microscale flows (see the reviews of [2–5]). Optimization strategies have also been of interest, but these strategies do not take into account coupling of the local heat-transfer problem with transfer to the environment outside the heat exchanger. Further, as the scales of the cross-sectional area become smaller, physical effects negligible on the macroscale can become relevant, such as viscous heating due to fluid flow for example (see [4]).

Although the experimental capability to design and fabricate specific microstructures in materials has advanced significantly in the past few decades, a description of their effective thermal behavior on the macroscale has not kept pace [6]. There are several technical challenges associated with modeling the energy transport in these systems. First, the length scales of the fundamental processes are several orders of magnitude smaller (100 nm–10 μm, or the *microscale*) than the scales dictated by a given application (1 mm–1 cm, or the *macroscale*) [7]. In addition, the rates of energy transport on the microscale need not be commensurate with the energy rates of transport on the macroscale. Further, multiple coupled physical processes, such as thermal, electromagnetic, mechanical and chemical thermodynamically interact under conditions which may be classically identified but whose net response from their interaction is not easily predicted.

Current modeling of these multi-physics systems has focused on direct computational approaches (see [8] as one example). With these studies, a prescribed design is tested through a variety of different flow conditions and coolant/power requirements. These *in silico* experiments are useful in evaluating a specific design, with the requirement that the periodic array of channels are identical over all of the channels. We are interested on the effect on global heat transport by slowly-varying changes in the geometry of the microchannels and the laminate thicknesses. The goal here is to develop a mathematical framework that captures the dominant physics and on which efficient computational design tools can be built.

The limitations of direct numerical simulation of these cooling systems is due to the need to resolve the finest length scale in the problem. For example the characteristic channel widths are on the order of tens of microns, but the device length scale is on the order of a centimeter. Hence, to directly compute the full

E-mail address: tilley@wpi.edu

0017-9310/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijheatmasstransfer.2013.02.035

Nomenclature

A	unknown spanwise temperature gradient (microscale)
Bi	Biot number
H	characteristic microscale length
L	characteristic macroscale length
ΔP^*	dimensional pressure scale
Pe	Peclet number $\Delta P^* H^2/(\mu_1 \kappa_1)$
Pr	Prandtl number of fluid
Re	Reynolds number of fluid
T	dimensional temperature
U	characteristic axial fluid velocity
c_n	arbitrary eigenfunction amplitude (mode n)
c_p	specific heat
h	channel or laminate thickness
k	thermal conductivity
$\hat{k}$	dimensionless geometric average of thermal conductivity
$\bar{k}$	dimensionless arithmetic average of thermal conductivity
$\mathbf{n}$	principle normal of local laminate/channel interface
p	fluid pressure
q	power source
$\mathbf{q}$	effective heat flux along chip boundary
t	time
u	fluid velocity component
$\mathbf{u}$	fluid velocity vector
x	macroscale coordinate
y	microscale coordinate

Greek Symbols

α	optimization parameter
δ	dimensionless ratio of streamwise convection to spanwise conduction
ϵ	aspect ratio H/L
κ	thermal diffusivity
η	eigenvalue of uniform laminate/channel thickness case
λ	dimensionless laminate-channel microscale period
ϕ	eigenfunction of uniform laminate/channel thickness case
ρ	mass density
μ	dynamic viscosity of fluid
θ	dimensionless temperature
$\bar{\theta}$	macroscale dimensionless temperature (independent of microscale)
ξ	integration variable

Subscripts

0,1,...	dependent variables: Correction of quantity to $O(\epsilon^{0,1,\cdots})$. Independent variables: 1 streamwise direction, 2 spanwise direction. Material properties: 1 fluid, 2 solid
ℓ	liquid
s	solid

Superscripts

$*$	dimensional quantity
(1)	dependent variable: fluid quantity
(2)	dependent variable: solid quantity

problem requires 10^6 more resources, either time or computational power. This separation of scales, or stiffness, limits not only the feasibility of a direct simulation but also the reliability of the resulting calculated solutions.

Effective models which capture the dominant physics in the microscale and their effect on macroscale dynamics are an attractive choice, since the mathematical description is computationally feasible, provides physical insight into the dominant macroscopic behaviors on the application time scales and operational frequencies, and their limits of validity are well understood (e.g. [9]). Classical asymptotic approaches to these problems require the material to be spatially periodic at a fixed wavenumber [10,11]. However, materials that are influenced by many physical processes do not conform to this modeling paradigm. We extend these asymptotic approaches here to allow for the spatial pattern wavenumber to vary over the macroscale variables in order to find how changes in microstructure geometry can affect macroscopic properties and transport. This extension then results in a problem of solving the flow field and thermal profile in a porous media (Ene and Polisevski [12] describe the classical homogenization approach). Liquid-cooled electronics provides a context in which these approaches can be implemented, and the results can be used to improve cooling system design.

We focus in this paper on a canonical problem of silicon laminates, in which energy is generated. Between the laminates are fluid channels which through which coolant flows to reduce the overall temperature of the system. Although this problem is geometrically simpler than systems found in applications, it has the benefit to formulate the mathematical problem completely and to better quantify the errors in the approximation. Further, in this paper we consider applications where power densities are "low" (at most 100 W/cm^2). Although limited in its description of the energy removal needs of current applications, this assumption simplifies the analysis significantly, and can directly describe most of the current generation of cooling systems. Note that this restriction then results in the assumptions of fully developed flow, fluid material properties that are independent of temperature, and viscous heating effects are ignored (see Rosa et al. [4] for a review of papers for single channels where these assumptions are relaxed). We discuss in the Conclusions section the challenges that need to be addressed to describe higher power applications, where these assumptions may not necessarily be made.

The outline of the paper is as follows. In Section 2 we outline the problem and derive a single partial differential equation for the temperature in the chip over the macroscale, which depends on the channel and laminate geometries, applied pressure drops and fluid and solid material parameters. In Section 3 we discuss the different modes of energy transport, how these modes depend on the channel-laminate geometry, and some examples of how the current modeling framework can be leveraged to find optimal channel patterns. We discuss our conclusions in Section 4.

2. Problem formulation

We consider the situation illustrated in Fig. 1. A series of heat-generating laminates are separated by a region filled with an incompressible liquid. The coolant is moved between the laminate by an applied pressure drop ΔP^*. The thickness of the solid and fluid regions scale on a length scale H, while the variation of the power generation in the laminates q^* varies on a much larger scale $L \gg H$. Our goal is to find an effective set of equations for the fluid pressure and the temperature based on the power distribution along the silicon laminates. Typical geometry and material properties of these systems are listed in Table 1.