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Isogeometric shape design optimization of heat conduction problems

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ABSTRACT

An isogeometric shape sensitivity analysis method is developed for heat conduction problems using the adjoint variable method. When the isogeometric method is extended to isogeometric shape optimization, the geometric properties of design are embedded in NURBS basis functions for CAD and response analysis so that the design parameterization is much easier than that in finite element based method without subsequent communications with CAD description since the perturbation of control points naturally results in shape changes. Thus, exact geometric models can be used in both response and shape sensitivity analyses, where normal vector and curvature are continuous over the whole design space so that enhanced shape sensitivity can be expected, especially for the heat conduction problems that include various types of boundary conditions. Moreover, the NURBS basis functions conveniently provide a smooth and non-local design velocity field for the shape design optimization, where computation is not easy in the finite element based method. Through numerical examples, the developed isogeometric sensitivity is verified to demonstrate excellent agreements with finite difference one. Also, it turns out that the proposed method works very well in various shape optimization problems.

1. Introduction

Typically in finite element based shape optimization, designs are embedded in CAD systems and finite element meshes are generated from the CAD data. The geometric approximation inherent in the mesh could lead to accuracy problems in response analysis and more adversely in design sensitivity analysis. Piecewise linear approximations of geometry turned out to be the root cause. Even though a mesh is constructed, further refinement requires communication with the CAD system during design iterations, which has been one of the major reasons that prevented the active industrial application of the shape optimization method.

The isogeometric analysis (IGA) based on NURBS (non-uniform rational B-splines) was developed by Hughes et al. [1] and Cottrell et al. [2]. The objective of the isogeometric analysis is to develop an analysis framework, employing the same basis functions as used in the CAD systems and embedding exact geometry. This isogeometric approach has many advantages: *first*, the geometric flexibility of the NURBS basis allows for the exact representation of geometry than any other approaches such as the standard finite element and meshfree methods. *Secondly*, subsequent refinement does not require any further communication with the CAD systems so that mesh refinement procedures are significantly simplified [3]. Besides the analogues of *h*-, *p*-, and *hp*-refinement in the standard finite element methods, the isogeometric approach provides a peculiar *k*-refinement capability that has advantages of efficiency and robustness over the traditional *p*-refinement. The isogeometric analysis method has many features in common with the finite element method; it invokes the isoparametric concept where the dependent variables and the geometry share the same basis functions. The basis functions directly come from the CAD model and are thus called "*isogeometric*." Also, the method has some features in common with the meshfree methods; it is not interpolatory. In this paper, a variational formulation for heat conduction problems is derived based on the isogeometric approach.

Among the various design sensitivity analysis (DSA) methods, the continuum-based adjoint variable method (AVM) is known to be the most efficient and accurate. The AVM is especially computationally efficient in case many design variables are involved and yet only a small number of performance measures are considered, because the design sensitivity can be computed in a selective manner [4]. In this study, we develop an efficient DSA method for the heat conduction problems in a steady state, and will discuss the continuum method for the boundary variations herein. There are very few studies conducted on the shape optimization of heat conduction problems. Tortorelli et al. derived the shape design sensitivity for nonlinear transient thermal systems using a Lagrange multiplier method [5] and the adjoint method [6]. Sluzalec et al. [7] employed the Kirchhoff transformation to derive the shape design sensitivity expressions for linearized heat conduction problems using an adjoint variable approach. Li et al. [8] performed a

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Nomenclature

h_c	convection coefficient	V_n	normal velocity
$\mathbf{n}$	outward normal vector	Y	trial function space
Q	internal heat generation	$\overline{Y}$	test function space
q	prescribed heat flux	Ω	domain
T	temperature field	Γ	boundary
T_0	prescribed temperature	γ	thermal conductivity
T_∞	ambient temperature	κ	curvature
$\boldsymbol{V}$	design velocity	λ	adjoint variable

shape and topology optimization of heat conduction problems using an evolutionary structural optimization method. Dems et al. derived the first-order sensitivity equation of heat conduction problems with respect to material property, external boundary, and internal interface and obtained an optimal design for various thermal problems [9] and steady conduction problems with radiation [10]. Cheng et al. [11] employed a body-fitted grid generation scheme to generate a curvilinear grid and determined the shape profiles of heat conduction problems by means of finite volume method. Ha and Cho [12] formulated a heat conduction problem using the level sets for versatile topological variations and performed topological shape optimization with various boundary conditions.

In addition to the benefits of the isogeometric analysis, the isogeometric design sensitivity analysis has the following advantages [13,14]: *firstly*, it provides more accurate sensitivity of complex geometries including higher order effects such as normal and curvature information. The NURBS functions of higher continuity offer a much more compact representation of the response and sensitivity of structures than the standard finite element functions do, yielding much greater accuracy, even at the same polynomial order. *Secondly*, it vastly simplifies the design modification of complex geometries without needing to communicate with the CAD description of geometry during the optimization process. Since the NURBS basic functions are used in the isogeometric response and sensitivity analyses, design modifications are easily obtainable using the adjustment of control points which represent the geometric model. In this paper, a continuum-based adjoint DSA method using the isogeometric approach is derived for the heat conduction problems. The conventional shape optimization methods using finite elements have some difficulties in design parameterization. In the isogeometric analysis, however, the geometric properties are already embedded into the NURBS basis functions and control points. The perturbation of the control points naturally results in shape changes. When using the conventional finite element method, the inter-element continuity of design space is not guaranteed and so the normal vector and curvature information are not accurate enough. On the other hand, in the isogeometric analysis, these values are continuous over the whole design space so that accurate shape sensitivity can be obtained.

Design velocity field, defined as a mapping rate between the original and the perturbed domains, is an important ingredient in computing shape design sensitivity coefficients and updating the finite element mesh in the process of shape design optimization. It is found that a combination of isoparametric mapping and boundary displacement methods is ideal for the computation of the design velocity field [15]. Arora et al. [16] discussed about the two basic methods of shape DSA, the material derivative and control volume approaches, for structural DSA to show that those were theoretically equivalent but numerically different in implementations. In the first approach, the material derivative concept is used to obtain the variations of field variables whereas, in the control volume approach, all of the quantities and integrals are transformed into a fixed reference domain and then the variations are taken to obtain the design sensitivity expressions afterwards.

The remainder of this paper is organized as follows, in section 2, we describe the construction of NURBS basis functions, which may have up to $(p-1)$ continuous derivatives across element boundaries where p is the order of the underlying polynomial. In section 3, we explain the isogeometric analysis framework based on the NURBS. We discuss the governing equation and weak formulation for the heat conduction problems. One important feature in the NURBS-based approach to isogeometric analysis is the capability to incorporate the functions of higher order and continuity. In section 4, we derive the isogeometric shape DSA expressions, where the geometric effects seem to have profound effects on shape sensitivity. The NURBS functions of higher continuity offer a much more compact representation of the shape sensitivity expressions. In section 5, we formulate the isogeometric shape optimization problems and present demonstrative numerical examples, where the accuracy of the isogeometric sensitivity is verified by the comparison with the finite difference one. *Finally*, we draw conclusions, which present the importance of the isogeometric approach.

2. NURBS basis function

2.1. B-spline basis function

In the isogeometric analysis, the solution space is represented in terms of the same basis functions as used in describing the geometry. The isogeometric analysis has several advantages over the conventional finite element analysis: *geometric exactness* and *simple refinements* due to the use of NURBS basis functions which are based on B-splines. Consider a knot vector $\boldsymbol{\Xi}$ in a one dimensional space, which includes the set of knots ξ_i in a parametric space.

$$\boldsymbol{\Xi} = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}, \tag{1}$$

where p and n are the order of the basis function and the number of control points, respectively. The B-spline basis functions are defined, recursively, as

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \leqslant \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases}, \quad (p = 0) \tag{2}$$

and

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p-1} - \xi_{i+1}} N_{i+1,p-1}(\xi), \quad (p = 1, 2, 3, \ldots). \tag{3}$$

Using the B-spline basis function $N_{i,p}(\xi)$ and weight w_i, the NURBS basis function $R_{i,p}(\xi)$ is defined as

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi) w_i}{\sum_{j=1}^{n} N_{j,p}(\xi) w_j}. \tag{4}$$