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Axisymmetric thermocapillary migration of a fluid sphere in a spherical cavity

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ABSTRACT

The quasi-steady thermocapillary migration of a spherical fluid drop situated at an arbitrary position in a second fluid within a spherical cavity is studied theoretically in the limit of negligible Marangoni and Reynolds numbers. The imposed temperature gradient is constant and along the line connecting the centers of the drop and cavity. To solve the thermal and hydrodynamic governing equations, the general solutions are constructed from the fundamental solutions in the two spherical coordinate systems based on the drop and cavity. The boundary conditions at the drop surface and cavity wall are satisfied by a collocation technique. Numerical results for the thermocapillary migration velocity of the drop normalized by its value in an unbounded medium are presented for various values of the relative viscosity and thermal conductivity of the drop, the relative conductivity of the cavity phase, the drop-to-cavity radius ratio, and the relative distance between the drop and cavity centers. In the particular case of the migration of a spherical drop in a concentric cavity, these results agree excellently with the exact solution derived analytically. The normalized thermocapillary migration velocity of the confined drop decreases monotonically with an increase in the drop-to-cavity radius ratio or its relative distance from the cavity center and vanishes in the touching limit of the drop and cavity surfaces. On the other hand, this velocity increases with an increase in the relative thermal conductivity of the cavity phase, but can increase or decrease with an increase in the relative viscosity or thermal conductivity of the drop for a given configuration. The boundary effect on thermocapillary migration can be significant, but is weaker than that on sedimentation.

1. Introduction

A small drop of one fluid suspended in a second fluid in which there is a temperature gradient will move toward the hotter side due to the temperature-induced interfacial tension gradient along the drop surface. This phenomenon, known as thermocapillary migration, plays an important role in material processing under microgravity condition and many other technological applications [1]. The thermocapillary migration of drops was first demonstrated by Young et al. [2], who observed the movement of gas bubbles in a vertical liquid bridge in the gap between the anvils of a micrometer. The lower anvil was heated to produce the temperature gradient to arrest the buoyant rise of the bubbles or even to drive the bubbles downward. They also derived a formula for the migration velocity $\mathbf{U}_0$ of a spherical drop of radius a present in an unbounded fluid of viscosity η with an imposed uniform temperature gradient ∇T_∞ in the limit of small Marangoni and Reynolds numbers,

$$\mathbf{U}_0 = \frac{2}{(2+k^*)(2+3\eta^*)}\left(-\frac{\partial\gamma}{\partial T}\right)\frac{a}{\eta}\nabla T_\infty. \quad (1)$$

In this formula, k^* and η^* are the ratios of thermal conductivities and viscosities, respectively, between the internal and ambient fluids, $\partial\gamma/\partial T$ is the variation of the interfacial tension γ at the drop surface with respect to the local temperature T, and all the physical properties are taken to be constant except for the interfacial tension, which is assumed to vary linearly with temperature. The thermocapillary mobility of a spherical gas bubble (with negligible thermal conductivity and viscosity in comparison with the surrounding liquid) can be evaluated by Eq. (1) taking the limiting values $k^* = 0$ and $\eta^* = 0$. According to Eq. (1), bubbles of radius 10 μm in water will migrate by thermocapillarity at a velocity about 0.7 mm/s in temperature gradients of order 1 K/mm.

Eq. (1) serves only for a fluid drop in continuous phases that extend to infinity in all directions. However, in practical applications of thermocapillary migration, drops usually are not isolated and will move in the presence of neighboring drops and/or boundaries [3–18]. During the past three decades, much progress has been made in the theoretical analysis concerning the applicability of Eq. (1) for the thermocapillary migration of a drop in a variety of

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Nomenclature

a radius of the drop, m
A_{1n}, A_{2n} coefficients in Eqs. (17) and (18) for the flow field, m^{-n+3} s^{-1}
A'_n, A''_n, A^*_n functions of position in Eqs. (17)–(20), m^{n-2}, m^{n-2}, m^{n-3}
b radius of the cavity, m
B_n coefficients in Eq. (18) for the flow field, m^{n+2} s^{-1}
B'_n, B''_n, B^*_n functions of position in Eqs. (17)–(20), m^{-n-1}, m^{-n-1}, m^{-n-2}
C_{1n}, C_{2n} coefficients in Eqs. (17) and (18) for the flow field, m^{-n+1} s^{-1}
C'_n, C''_n, C^*_n functions of position in Eqs. (17)–(20), m^n, m^n, m^{n-1}
d distance between the centers of the drop and cavity, m
D_n coefficients in Eq. (18) for the flow field, m^n s^{-1}
D'_n, D''_n, D^*_n functions of position in Eqs. (17)–(20), m^{-n+1}, m^{-n+1}, m^{-n}
$\mathbf{e}_z$ unit vector in z direction
E_∞ $=|\nabla T_\infty|$, K m^{-1}
F force acting on the drop, N
k thermal conductivity of the external fluid, W m^{-1} K^{-1}
$\hat{k}$ thermal conductivity of the drop, W m^{-1} K^{-1}
k_w thermal conductivity of the cavity phase, W m^{-1} K^{-1}
k^* $=\hat{k}/k$
k^*_w $=k/k_w$
M, N numbers of collocation points on the drop and cavity surfaces
p dynamic pressure in the external fluid, Pa
$\hat{p}$ dynamic pressure in the drop, Pa
P_m Legendre function of order m
r_1 radial spherical coordinate based on the center of the drop, m
r_2 radial spherical coordinate based on the center of the cavity, m
R_{1m}, R_{2m} coefficients in Eqs. (8) and (9) for the temperature field, m^{-m+1}
S_{1m}, S_{2m} coefficients in Eqs. (9) and (10) for the temperature field, m^{m+2}
T temperature field in the external fluid, K
$\hat{T}$ temperature field in the drop, K
T_w temperature field in the cavity phase, K
T_0 undisturbed temperature at the center of the drop, K
T_∞ prescribed temperature field defined by Eq. (7), K
U velocity of the drop, m s^{-1}
$\mathbf{U}_0$, U_0 velocity of an isolated drop, m s^{-1}
$\mathbf{v}$ velocity field in the external fluid, m s^{-1}
$\hat{\mathbf{v}}$ velocity field in the drop, m s^{-1}
v_ρ, v_z components of $\mathbf{v}$ in cylindrical coordinates, m s^{-1}
$\hat{v}_\rho$, $\hat{v}_z$ components of $\hat{\mathbf{v}}$ in cylindrical coordinates, m s^{-1}
z axial cylindrical coordinate, m

Greek symbols
γ interfacial tension, kg s^{-2}
$\delta^{(1)}_m$, $\delta^{(2)}_m$ functions of position defined by Eqs. (A1) and (A2), m^{m-1}, m^{-m-2}
$\delta^{(3)}_m$, $\delta^{(4)}_m$ functions of position defined by Eqs. (A3) and (A4), m^{-m-2}, m^{m-1}
η viscosity of the external fluid, kg m^{-1} s^{-1}
$\hat{\eta}$ viscosity of the drop, kg m^{-1} s^{-1}
η^* $=\hat{\eta}/\eta$
θ_1, ϕ angular spherical coordinates based on the center of the drop
θ_2, ϕ angular spherical coordinates based on the center of the cavity
λ $=a/b$
ρ radial cylindrical coordinate, m
$\tau_{r\theta}$ viscous shear stress in the external fluid, Pa
$\hat{\tau}_{r\theta}$ viscous shear stress in the drop, Pa

bounded systems. Using spherical bipolar coordinates, Meyyappan et al. [19] and Sadhal [20] semi-analytically determined the quasi-steady thermocapillary migration velocity of a spherical gas bubble perpendicular to an infinite plane surface of constant temperature, and Meyyappan and Subramanian [21] investigated the thermocapillary motion of a gas bubble parallel to a plane wall prescribed with a linear temperature distribution. Later, Barton and Subramanian [22] and Chen and Keh [23] also used spherical bipolar coordinates to solve the problem of thermocapillary migration of a fluid drop normal to an isothermal plane surface. Analytical solutions of this problem in asymptotic forms were also obtained by using a method of reflections [23] and a lubrication theory [24]. In the aforementioned thermocapillary motions of a bubble or drop near a plane boundary, the migration velocity was found to decrease monotonically as the drop approaches the boundary as a result of the drop-boundary thermal and hydrodynamic interactions.

The thermocapillary migration of a fluid sphere within narrow pores has also been examined. Using a boundary collocation method, Chen et al. [25] solved for the thermocapillary mobility of a spherical drop along the axis of an insulated circular tube, which is a monotonically decreasing function of the drop-to-tube radius ratio and increases as k^* decreases, because a greater portion of energy is conducted through the gap between the less conductive drop and the insulated tube wall creating larger interfacial tension gradients at the drop surface. On the other hand, the thermocapillary motions of a spherical drop parallel [26] and perpendicular [27] to two plane walls at an arbitrary position between them have been investigated by using both the boundary collocation technique and the method of reflections. The drop velocity parallel to one or two plane walls prescribed with a linear temperature profile increases as k^* increases. Under the condition of large values of k^* and η^*, the thermocapillary mobility of the drop can increase with a decrease in the drop-to-wall distance and even be greater than its value in an unbounded medium given by Eq. (1) when the gap between the drop and the wall becomes thin.

In addition to the investigations of boundary effects on the thermocapillary migration of fluid spheres, the motions of a deformable drop near another drop [28], normal to an isothermal plane wall [29], and in a square box [30] due to thermocapillarity have been studied. The boundary effects on the thermocapillary migration of fluid drops normal to a plane wall [31] and the interaction in thermocapillary motion of two drops with different diameters [32] have also been examined experimentally and were found to be consistent with the theoretical predictions. In all of the previous studies of boundary effects on thermocapillary migration of fluid drops, the effect of the thermal conductivity of the boundary phase has never been considered.

Microfluidic systems with mazes of microchannels along which drops conveying solutes or materials undergo thermocapillary motion are coming into use to perform bio/physico-chemical analyses or to produce novel entities [33]. The system of a fluid sphere moving inside a spherical cavity can be taken as an