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Inverse hyperbolic thermoelastic analysis of a functionally graded hollow circular cylinder in estimating surface heat flux and thermal stresses

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ABSTRACT

In this study, an inverse algorithm based on the conjugate gradient method and the discrepancy principle is applied to solve the inverse hyperbolic heat conduction problem in estimating the unknown time-dependent inner-wall heat flux of a hollow cylinder from the knowledge of temperature measurements taken within the medium. The inverse solutions have been justified through the numerical experiments in two specific cases to determine the unknown heat flux. Temperature data obtained from the direct problem are used to simulate the temperature measurements. The influence of measurement errors upon the precision of the estimated results is also investigated. Results show that excellent estimation on the time-dependent heat flux can be obtained for the test cases considered in this study. Once heat flux variation is accurately estimated, the evolution of temperature, displacement, and stress distributions can be calculated in great precision.

1. Introduction

Functionally graded materials (FGMs), originally proposed by Japanese researchers [1], are a brand of composite materials which are designed to optimize the stress and thermal resistance and temperature distribution in components or structures subjected to extreme thermal or mechanical loading. Unlike traditional laminated composite materials where several layers of different materials are stuck together, the transition between different materials is made gradually in order to reduce the local stress concentration induced by abrupt changes in material properties. In recent years, FGMs have even been proposed as a solution for aerospace industry where temperature resistant, light-weight structures are required to meet the challenges faced by future high-speed space vehicles. These novel materials have excellent thermo-mechanical properties to withstand high temperature and have been extensively applied to important structures, such as nuclear reactors, pressure vessels and pipes, chemical plants, etc. [2–4]. For example, a thin functionally graded thermal shield can sustain steep temperature gradients without excessive thermal stresses. Similar advantages can be realized with functionally graded heat exchanger pipes and heat engine components, in which FGMs that continuously transit from ceramic to metallic materials would avoid the mismatch of the thermal expansion coefficient found in laminated materials.

Fourier's law has been traditionally the mainstream theory used to solve heat conduction problems. Although Fourier's law bears a theoretical flaw that thermal signal travels at an infinite speed, it solves most large time and/or length scales engineering heat conduction problems with satisfactory accuracy. Yet the development of laser heating and nanotechnology has created heat conduction problems within very small time and/or length scales. For example, a carefully controlled incident beam can be used to heat up a very small patch of area at a rate up to 180 K/s for a few nanoseconds [5]. In such situations, researchers have reported that the predictions by Fourier heat conduction do not agree well with experimental observations. Maurer and Thompson [6] observed the surface temperature of a slab taken immediately after a sudden thermal shock is 300 K higher than that predicted by Fourier's law. The disagreement between Fourier prediction and such experimental observation is rooted in the unrealistic propagation speed of thermal signal adopted by Fourier's law. In reality, a thermal signal travels at a finite speed, making a thermal response to behave like a wave. To better describe this wave-like behavior, instead of using Fourier's law, the Maxwell–Cattaneo equation, which takes finite thermal signal travelling speed into account, can be used. This approach, however, has led to a more complicated hyperbolic governing equation on heat conduction problems. Because FGMs are usually subjected to high temperature, a precise evaluation of their thermal characteristics is of great interest for engineering design and manufacture [7]. Therefore, hyperbolic heat conduction in FGMs is an interesting subject to study. However, the complexity of solving hyperbolic heat conduction in FGMs analytically or

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Nomenclature

E	elastic modulus (GPa)
h	convection heat transfer coefficient (W m^{-2} K^{-1})
J	functional
J'	gradient of functional
k	thermal conductivity (W m^{-1} K^{-1})
p	direction of descent
q	heat flux at the inner surface (W m^{-2})
r	space coordinate in r-direction (m)
r_1	inner radius of the cylinder (m)
r_2	outer radius of the cylinder (m)
T	temperature (K)
T_r	reference temperature (K)
T_∞	outer ambient temperature (K)
t	time (s)
u	displacement radial component (m)
Greek symbols	
Δ	small variation quality
α	thermal diffusivity (m^2 s^{-1})
β	step size
γ	conjugate coefficient
ε	very small value
λ	variable used in the adjoint problem
ξ	transformed time coordinate
ρ	density (kg m^{-3})
σ	standard deviation
σ_r	stress radial component (GPa)
σ_z	stress axial component (GPa)
σ_θ	stress tangential component (GPa)
τ	relaxation time (s)
ω	thermal expansion coefficient (1/K)
ϖ	random variable
Superscripts	
K	iterative number
$*$	dimensionless quantity

numerically presents a difficult challenge, and only very few attempts to do so have been documented in open literature [8–10].

Quantitative studies of heat transfer processes occurring in many industrial applications often require accurate knowledge of boundary conditions, such as heat flux, or thermophysical properties of the materials involved. These important quantities were conventionally obtained by expensive experimental methods which normally involve delicate and sophisticated equipments. In recent years, however, the studies of inverse heat conduction problem (IHCP) have offered convenient alternatives, which largely scale down experimental work, to obtain accurate thermophysical quantities such as heat sources, material's thermal properties, and boundary temperature or heat flux distributions, in many heat conduction problems. To date, a variety of analytical and numerical techniques have been developed for the solution of the inverse heat conduction problems, for example, the conjugate gradient method (CGM) [11–13], the genetic algorithm [14], and the linear least-squares error method [15], etc.

Although there have been a great number of reports dealing with the inverse solutions of classical Fourier heat conduction problems, the reports on inverse hyperbolic heat conduction problems have been much fewer in open literature. Huang and Wu [16] studied the inverse hyperbolic problem of a straight fin by an iterative regularization method in estimating the unknown base temperature based on the boundary temperature measurements. Yang [17] proposed a sequential method for estimating the boundary conditions in a two-dimensional hyperbolic heat conduction problem. Das et al. [18] estimated the extinction coefficient and the conduction-radiation parameter simultaneously in a non-Fourier conduction and radiation heat transfer problem by the genetic algorithm in combination with the lattice Boltzmann method and the finite-volume method. Yet, as far as applying an inverse analysis on hyperbolic heat conduction in FGMs is concerned, to the best of the authors' knowledge, there has not been such a report in open literature.

Many engineering components assume the shape of a hollow cylinder. The inverse analysis of the heat transfer process of a hollow cylinder allows accurate measurements to be taken with much less experimental effort and sheds light into the heat transfer process and the reactions of the material's stresses to thermal loading. Therefore, the focus of the present study is to develop an inverse hyperbolic analysis for estimating the unknown time-dependent heat flux at the inner wall of a hollow cylinder constructed with FGMs from the knowledge of temperature measurements taken within the medium. To this end, we present the conjugate gradient method and the discrepancy principle [19] to estimate the unknown time-dependent heat flux by using the simulated temperature measurements. The CGM derives from the perturbation principles and transforms the inverse problem to the solution of three problems, namely, the direct, sensitivity and the adjoint problem, which will be discussed in detail in the following sections. On the other hand, the first-and second-derivatives in all governing equations in the three problems are discretized by backward differencing to avoid adding the tedious Laplace inverse transform on top of this already complicated inverse process.

2. Analysis

2.1. Direct problem

To illustrate the methodology of developing expressions for the use in determining the unknown time-dependent heat flux at the inner surface of a functionally graded hollow cylinder in an inverse

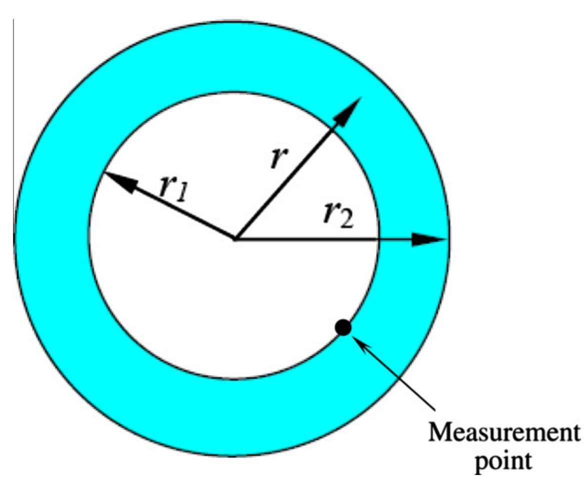

Fig. 1. Geometry and coordinate system.