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Non-Fourier bio heat transfer modelling of thermal damage during retinal laser irradiation

Arunn Narasimhan *, Sridhar Sadasivam *

Heat Transfer and Thermal Power Laboratory, Department of Mechanical Engineering, Indian Institute of Technology Madras, Chennai 600 036, India

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ABSTRACT

Study of non-Fourier energy transport is important in various biomedical applications, in the understanding of heat transfer in biological tissues. One such biomedical application that involves non-Fourier heat conduction is laser irradiation of the retina, where a high heat flux is typically applied for a short period of time. In the present study, retinal laser irradiation is analysed using the dual phase lag model for heat conduction. A simplified one-dimensional model of the human eye with seven layers is used in the simulation. The laser heating is modelled as a volumetric heat source and the respective magnitudes are calculated based on the absorptivities of the various layers. Apart from the temperature distribution, the damage distribution is also computed using the Arrhenius damage integral approach. The effect of the two phase lags τ_q and τ_T on the temperature and damage distributions are analysed. Small τ_T/τ_q ratios ($\leqslant 0.01$) indicate a thermal wave like behaviour, which diminishes as the ratio increases (i.e. for $\tau_T/\tau_q \geqslant 0.1$). The results are also compared with corresponding results from the Fourier model. Choroidal blood perfusion rate is found to have no significant effect on the unsteady temperature and damage distributions.

1. Introduction

Lasers are now being extensively used in ophthalmology to treat a variety of ocular disorders. Laser surgery is used to treat maladies such as diabetes-induced abnormal blood vessel growth, glaucoma and retinal tears, in addition to routine treatments such as vision correction. For effective laser surgical processes, it is important for an ophthalmologist to understand the evolution of temperature in the eye as a result of material and heat transfer parameters that affect the laser treatment. Mathematical models are useful in this regard as experimental methods are difficult and at times unsafe to perform in the human eye undergoing surgery.

One of the earliest modelling studies was reported in [1], which used finite different algorithm to solve Maxwell's equations to understand the effect of microwave irradiation on the human eye. Scott [2] used the Galerkin finite element method to analyse the variation of temperature in the frontal portion of the eye by considering evaporation from the corneal surface and blink factors as parameters. Flyckt et al. [3] studied the impact of blood flow on the temperature distribution. Three different methods were considered for modelling the cooling effect of blood flow-using a simple heat transfer coefficient at the sclera, using the Pennes bio-heat transfer equation and using the discrete vasculature model of the blood vessels. Narasimhan et al. [4] used the finite volume method to perform two dimensional transient simulations of retinal eye surgery. In [5] three dimensional multi-spot simulations of the retinal laser surgery was reported. They arrived at a minimum value of spot spacing required to reduce inter-spot interaction and hence overheating.

In an early paper that discussed non-Fourier heat conduction, Cattaneo [6] proposed a modification to the Fourier's law by introducing a time delay on the heat flux. A wave-like equation for temperature was formulated. This came to be known as the Cattaneo–Vernotte heat conduction equation or the hyperbolic heat conduction equation. This hyperbolic equation was later extended by Tzou [7] who introduced a time delay on the temperature gradient and the constitutive relation was then used to derive the dual phase lag (DPL) heat conduction equation.

Although the dual phase lag equations were derived, they were believed to be applicable only for high flux heating over very small time scales. However [8] reported experimental results on processed meat, which suggested the existence of non-Fourier heat conduction in non-homogenous materials as well. They reported a thermal relaxation value of about 15 s, which is much larger than those reported for metals with pico-second relaxation times. Such a high value of the relaxation time is a result of the microstructure of non-homogenous materials as found in meat and biological tissues.

* Corresponding authors.
E-mail address: arunn@iitm.ac.in (A. Narasimhan).

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Nomenclature

A	prefactor (s^{-1})
c_p	specific heat of tissue ($J\ kg^{-1}\ K^{-1}$)
D	length (m)
E_a	activation energy ($J\ mol^{-1}$)
h	heat transfer coefficient ($W\ m^{-2}\ K^{-1}$)
k	thermal conductivity of tissue ($W\ m^{-1}\ K^{-1}$)
$\mathbf{q}$	heat flux ($W\ m^{-2}$)
Q	volumetric heat source ($W\ m^{-3}$)
R	universal gas constant ($J\ mol^{-1}\ K^{-1}$)
RPE	Retinal Pigmented Epithelium
T	temperature of tissue (K)
w	perfusion rate (s^{-1})

Greek symbols

α	thermal diffusivity of tissue ($m^2\ s^{-1}$)
η	non dimensional space coordinate
γ	absorptivity
Ω	damage
ρ	density of tissue ($kg\ m^{-3}$)
τ_q	phase lag on heat flux (s)
τ_T	phase lag on temperature gradient (s)
θ	non dimensional temperature

Subscripts

a	ambient
b	blood
l	laser
m	metabolic

Non-Fourier bio-heat transfer models have been applied to understand practical, real-life situations. Zhou et al. [9,10] used the DPL model to study laser-induced thermal damage to biological tissues. The DPL model was found to produce temperature and thermal damage results that are significantly different from those obtained using the Fourier model. Liu and Chen [11] applied the bio-Pennes model to study magnetic hyperthermia treatment and understand the influence of the relaxation times τ_q and τ_T on the treatment parameters. An important feature of all these studies is that no specific values for the relaxation times τ_q and τ_T were used; they were instead considered as parameters and their values were varied over the range of values reported in literature.

A survey of literature for numerical models of laser eye surgery shows the uniform use of Fourier heat conduction model to study heat transfer processes in the eye. However many laser treatment processes that are similar to the retinal eye surgery discussed in [4,5] involve high heat fluxes over very small time scales in the order of milliseconds. However the relaxation time of biological tissues has been estimated to be of the order of 1–100 s [8,12–14]. Since the time scales of many laser eye treatments are much smaller than the relaxation time, the consideration of non-Fourier effects in energy transport becomes important.

In this paper, a dual phase lag constitutive relation along with the Pennes bio heat transfer model is used to develop a one-dimensional model of the retinal eye surgery discussed in [4,5]. The Arrhenius integral approach is used to compute the thermal damage as discussed in [15]. The effect of relaxation times τ_q and τ_T on temperature distribution and thermal damage during the retinal laser surgery is studied through numerical simulations.

2. Mathematical modelling and numerical method

2.1. Bio-Pennes dual phase lag model

The dual phase lag constitutive relation proposed by in [7] is shown below.

$$\mathbf{q} + \tau_q \frac{\partial \mathbf{q}}{\partial t} = -k\nabla T - k\tau_T \frac{\partial \nabla T}{\partial t} \tag{1}$$

Heat transfer in biological tissues is modelled using the Pennes equation, which includes an additional blood perfusion term to model heat transfer between blood and the tissue. The bio-Pennes heat transfer equation is given below.

$$\rho c_p \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} + Q_m + Q_l + (w\rho c)_b (T_b - T) \tag{2}$$

where Q_m and Q_l represent in $W\ m^{-3}$ the volumetric heat generation due to metabolism and laser heat energy. The last term on the RHS represents the heat transfer due to blood perfusion between the blood and the tissue where w_b denotes the blood perfusion rate.

Eq. (2) combined with (1) represents the mathematical model for our analysis of heat transfer in the human eye. Combining Eqs. (2) and (1), the following equation is obtained in terms of heat flux.

$$\tau_q \frac{\partial^2 \mathbf{q}}{\partial t^2} + \frac{\partial \mathbf{q}}{\partial t} = \alpha \Bigg(\nabla(\nabla\cdot\mathbf{q}) + \tau_T \frac{\partial}{\partial t}[\nabla(\nabla\cdot\mathbf{q})] - \nabla Q_l - \tau_T \left[\frac{\partial}{\partial t}(\nabla Q_l)\right] + \left((w\rho c)_b (\nabla T - \nabla T_b) + \tau_T (w\rho c)_b \left[\frac{\partial}{\partial t}(\nabla T - \nabla T_b)\right]\right)\Bigg) \tag{3}$$

In the present study, Eq. (3) is solved simultaneously with Eq. (2) to obtain temperature distributions during the laser surgical process. In addition to the choroidal blood perfusion, a constant convection heat transfer coefficient of 65 $W\ m^{-2}\ K^{-1}$ was assumed at sclera to include the influence of blood flow at the back of sclera. A constant heat transfer coefficient of 20 $W\ m^{-2}\ K^{-1}$ at the cornea was also included to account for convection, evaporation and radiation based heat transfer at that region. Except for the validation studies, the initial temperature gradient across the eye domain was set to be the steady state temperature variation obtained through simulations performed between 37 °C at the sclera and 42° at the corneal interface with the atmosphere. Heat flux continuity is assumed to prevail at the regional interfaces of the eye domain.

Apart from temperature distribution, damage distribution is also computed using the Arrhenius damage integral approach explained in [15]. The rate of change of damage Ω is given by the following expression.

$$\frac{d\Omega}{dt} = A \exp\left(\frac{-E_a}{RT(t)}\right) \tag{4}$$

where A is the prefactor, E_a is the activation energy for the damage process, R is the universal gas constant and T is the temperature of the tissue. The Arrhenius damage integral is based on the assumption that the pathway that leads to retinal damage is a zero order chemical reaction [16]. Additionally, according to Eq. (4), there is a damage even at normal body temperatures as $d\Omega/dt > 0$ for any temperature T. However this is not true in reality, as damage is observed to occur only at elevated temperatures [16]. Despite the