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Flow and heat transfer in a nano-liquid film over an unsteady stretching surface

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ABSTRACT

The unsteady film flow and heat transfer of nanofluids caused by a linear stretching velocity over a horizontal elastic sheet is investigated. By means of similarity variables, the boundary layer equations describing the momentum and energy conservations are reduced to a set of ordinary differential equations with an unknown constant. The homotopy analysis method (HAM) is then applied to give exact solutions for this particular kind of equations. A linear relationship between the film thickness β and the unsteadiness parameter S is found. Besides, the effects of the unsteadiness parameters S, the solid volume fraction of the nanofluid ϕ and the Prandtl number Pr on the velocity and the temperature distributions are presented and discussed, respectively. It is found that the usage of nanoparticles in the base fluids can effectively improve on the heat transfer characteristics.

1. Introduction

The study of laminar boundary layer flow over a stretching sheet has received considerable attention in the past due to its applications in the industries. The thermal processing of sheet-like materials is a necessary operation in the production of paper, linoleum, polymeric sheets, roofing shingles, insulating materials, fine-fiber mattes, boundary layer along a liquid film in condensation processes, etc. [1]. In virtually all such processing operations, the sheet moves parallel to its own plane. The moving sheet may induce motion in the neighboring fluid or, alternatively, the fluid may have an independent forced-convection motion that is parallel to that of the sheet. The aim in any extrusion process is to achieve a good quality surface of the extrudate. It is very important to control the drag and the heat flux for better product quality. The study of laminar flow of a thin liquid film over stretching sheet is currently attracting the attention of a growing number of researchers because of the immense potential of nanofluids to be used as technological tools in many engineering applications. The main application of such flows is in coating process such as in wire and fiber coatings. Other applications can be found in food processing, transpiration cooling, reactor fluidization and so on. All coating processes demand a smooth glossy finish to meet the requirements for best appearance and optimum performance such as low friction, transparency and strength. The rate of heat and mass transfer within the thin liquid film has a direct bearing on the success of the coating process and the chemical characteristics of the product. In view of these applications, Wang [2] initiated the study of hydrodynamics of thin liquid film over a stretching sheet by reducing the unsteady Navier–Stokes equations to non-linear ordinary differential equations by means of similarity transformations and solved the resulting differential equations using a multiple shooting method (see Robert and Shipman [3]). Wang's [2] work was further extended by different researchers with consideration of various velocity and thermal boundary conditions, such as, Usha and Sridharan [4], Andersson et al. [5], Wang [6], Dandapat et al. [7–9], Chen [10,11], Liu and Andersson [12,13], and Abbas et al. [14].

All the above investigations were restricted to the laminar flow of Newtonian (pure) fluids. However, in the recent past, nanofluids (a term proposed by Choi [15]) have attracted the attention of the science and engineering community because of their possible applications in industries. Nanotechnology is an emerging science that is extensive use in industry due to the unique chemical and physical properties that nano-sized materials possess. These fluids are colloidal suspensions, typically metals, oxides, carbides, or carbon nanotubes in a base fluid, etc. Common base fluids include water and ethylene glycol. Nanofluids have properties that make them potentially useful in many heat transfer processes including microelectronics, fuel cells, pharmaceutical processes and hybrid-powered engines. They exhibit enhanced thermal conductivities and heat transfer coefficients compared to the base fluids. For this reason nanofluids can often be preferred to conventional coolants like oil, water and ethylene glycol mixtures. The comprehensive

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references on nanofluid can be found in the recent book by Das et al. [16] and in the review papers by Buongiorno [17], Daungthongsuk and Wongwises [18], Ding et al. [19], Wang and Mujumdar [20,21], and Kakaç and Pramuanjaroenkij [22]. Particularly, Eastman et al. [23] and Xie et al. [24] showed that higher thermal conductivity can be achieved in thermal systems using nanofluids. Researchers have tried to increase the thermal conductivity of base fluids by suspending micro- or larger-sized solid particles in fluids, since the thermal conductivity of solid is typically higher than that of liquids, as seen from Table 1 [25]. Since nanofluid consists of very small sized solid particles, therefore in low solid concentration it is reasonable to consider nanofluid as a single phase flow [26].

The present paper is intended to analyze the unsteady nano-liquid film caused by a linear stretching velocity over a horizontal elastic sheet by using the mathematical nanofluid model described by Tiwari and Das [27]. With this model, several studies on the boundary layer flow and heat transfer have been available in the literature, as shown in [30–32]. Three kind of nanofluids, namely, Copper–water, Titanium dioxide–water and Aluminum oxide–water nanofluids are considered. The similarity solutions on momentum and energy equations are formulated via a set of similarity variables. The HAM technique is then applied to give exact solutions to the resulting ordinary differential equations. Besides, the effects of the unsteadiness parameters S, the solid volume fraction of the nanofluid ϕ and the Prandtl number Pr on the velocity and the temperature distributions are presented and discussed. Two main approaches have been adopted in the literature to investigate the heat transfer enhancement by small solid particles (millimeter and/or micrometer-sized particles) suspended in a fluid. The first approach is the two-phase model, which enables a better understanding of both the fluid and the solid phases role in the heat transfer process. The second approach is the singlephase model in which both the fluid phase and the particles are in thermal equilibrium state and flow with the same local velocity [33]. On the other hand, it should be mentioned at this end, that the study of nanofluids is still at its early stage, and it seems very difficult to have a precise idea on the way the use of nanoparticles acts in fluid flow and heat transfer, and complementary works are needed to understand the heat transfer characteristics of nanofluids and identify new and unique applications for these fluids [34].

2. Basic equations

We consider a nano-liquid film flow and heat transfer in the vicinity of a thin elastic sheet. The physical sketch is plotted in Fig. 1. Here the Cartesian coordinate system (x,y) is chosen such that the x-axis is measured in the direction of wall stretching and the y-axis is normal to the wall. The continuous surface at $y = 0$ is stretched with the velocity

$$U_w = \frac{bx}{1-\alpha t}, \tag{1}$$

where b and α are constants with dimensions time^{-1}. The temperature distribution on the sheet is given by

$$T_s = T_0 - T_r\left(\frac{bx^2}{2\nu_f}\right)(1-\alpha t)^{-3/2}, \tag{2}$$

where T_0 is the temperature at the slit, T_r can be taken either as a constant reference temperature or a constant temperature difference, ν_f is the kinematic viscosity of the pure fluid.

With assumptions that the film is uniform and stable and the end effects and the gravity are negligible, the governing equations for this problem are, by means of the mathematical model suggested by Tiwari and Das [27], written as

Table 1
The dimensionless film thickness β for various values of S and ϕ with $Pr = 1$.

Types of fluids	S	$\phi = 0.0$	$\phi = 0.1$	$\phi = 0.2$
Cu–water	0.6	3.131710299	2.665861561	2.571098243
	0.8	2.151993715	1.831879977	1.766762162
	1.0	1.543616057	1.313999817	1.267291082
	1.2	1.127780943	0.960021079	0.925895222
	1.4	0.821032220	0.698901895	0.674058038
	1.6	0.576173019	0.490466031	0.473031441
	1.8	0.356388848	0.303375233	0.292591157
Al_2O_3–water	0.6	3.131710299	3.135560807	3.276125841
	0.8	2.151993715	2.154639639	2.251230652
	1.0	1.543616057	1.545513968	1.614798296
	1.2	1.127780943	1.129167575	1.179787380
	1.4	0.821032220	0.822041698	0.858893261
	1.6	0.576173019	0.576881437	0.602742635
	1.8	0.356388848	0.356827036	0.372823346
TiO_2–water	0.6	3.131710299	3.102187358	3.219971442
	0.8	2.151993715	2.131706659	2.212643458
	1.0	1.543616057	1.529064237	1.587119863
	1.2	1.127780943	1.117149241	1.159565248
	1.4	0.821032220	0.813292269	0.844171411
	1.6	0.576173019	0.570741379	0.592411331
	1.8	0.356388848	0.353029135	0.366432972

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \tag{4}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

where u, v are the velocity components along x- and y-axis, T is the temperature, μ_{nf}, ρ_{nf} and α_{nf} are, respectively, the viscosity, the density and the thermal diffusivity of nanofluid, which are defined by

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{5/2}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \tag{6}$$

with

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \quad \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}. \tag{7}$$

Here ϕ is the solid volume fraction of the nanofluid, k_{nf} and $(\rho C_p)_{nf}$ are, respectively, the thermal conductivity and the heat capacitance of the nanofluid, ρ_f, μ_f, k_f and $(\rho C_p)_f$ are, respectively, the density, the dynamic viscosity, the thermal conductivity and the heat capacitance of the base fluid, ρ_s, μ_s, k_s and $(\rho C_p)_s$ are, respectively, the density, the dynamic viscosity, the thermal conductivity and the heat capacitance of the nanoparticle.

The associated boundary conditions for Eqs. (3)–(5) are

$$u = U_w, \quad v = 0, \quad T = T_s, \quad \text{at } y = 0, \tag{8}$$

$$\frac{\partial u}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = 0, \quad v = \frac{dh}{dt} \quad \text{at } y = h(t), \tag{9}$$

where $h(t)$ is the film thickness. Here $x \geqslant 0$ is assumed.

In the boundary layer approximations (see Schlichting and Gersten [28]), the boundary-layer thickness $\delta(x)$ is proportional to $(x\nu_f/U_w)^{1/2}$, so that the similarity variable η is set as

$$\eta = y\sqrt{\frac{U_w}{\nu_f x}} = y\sqrt{\frac{b}{(1-\alpha t)\nu_f}}. \tag{10}$$

The above relationship can also be obtained by the definitions proposed by Görtler [29] for similarity transformations. With his definitions, we have