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Laminar natural convection in an inclined cylindrical enclosure having finite thickness walls

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ABSTRACT

Mathematical simulation of unsteady natural convection in an inclined cylinder with heat-conducting walls of finite thickness and a local heat source in conditions of convective heat exchange with an environment has been carried out. Numerical analysis has been based on solution of the convection equations in the dimensionless variables vector potential components, modified vorticity functions, temperature. Particular efforts have been focused on the effects of four types of influential factors such as the Rayleigh number $Ra = 10^4,\ 5\cdot 10^4,\ 10^5$, the Prandtl number $Pr = 0.7,\ 7.0$, the thermal conductivity ratio $k_{2,1} = 5.7\cdot 10^{-4},\ 4.3\cdot 10^{-2}$ and the inclination angle $\gamma = 0,\ \pi/6,\ \pi/3,\ \pi/2$ on the velocity and temperature fields. The effect scales of the key parameters on the average Nusselt number have been determined.

1. Introduction

Study of natural convection is related to definition of optimum heat transfer modes in various technological systems such as heat pipes [1,2], thermosyphons [3,4], cooling systems of heat-generating components in electronics [5,6], chemical reactors [7]. Correct definition of the most effective conditions of the transport processes evolution in such devices is possible only by multiparameter mathematical simulation of nonstationary convective heat transfer modes [8]. To date, a bundle of experimental and theoretical studies of natural convection regimes in cavities with various shapes [9–26] has been conducted. The majority of the investigations concern the numerical analysis of transport processes in the two-dimensional objects both in view of heat-conducting walls effect [11–15], and in case of absence of such influence [16–19]. Thus it is necessary to note essential differences of the obtained results [12,13,15], that is caused by significant thermal lag effect of solid walls. For example, Liaqat and Baytas [13] have numerically analyzed natural convection in an enclosure having both heat-conducting walls of finite thickness and without them. The obtained results reflect strong effect of heat-conducting walls of finite thickness on heat transfer regimes. Sheremet [15] has analyzed the diffusion effects in the conjugate heat and mass transfer problems in a wide range of key parameters. It was shown that essential changes of thermohydrodynamic regimes in a cavity and also a significant decrease in the average Nusselt and Sherwood numbers is observed in case of infinitely thin walls.

The effect of heat-conducting walls on the velocity and temperature fields has been pointed out for the three-dimensional natural convection regimes in rectangular domains [21–23]. Kuznetsov and Sheremet [21] have shown that for the conjugate Rayleigh–Benard problem a decrease in the thermal conductivity ratio leads both to an increase in temperature in a cavity and to a reduction in the average Nusselt number. Valencia et al. [22] have conducted the experimental and numerical analysis of natural convection in a cubical enclosure with and without heat-conducting walls of finite thickness at $10^7 \leqslant Ra \leqslant 10^8$. It has been shown, that in case of the conjugate problem the change in circulation rates and temperature of a fluid in the cavity is observed. Ha and Jung [23] have numerically investigated the effect of the heat-generating cubic conducting body on the flow structure in a vertical cubic enclosure. These authors demonstrated that the presence of the heat-conducting body leads to a significant change in the average Nusselt number.

The effect of heat-conducting walls on the velocity and temperature fields has been unfairly neglected at the analysis of three-dimensional convective heat transfer in cylindrical enclosures [24–26]. Li et al. [24] have numerically analyzed unsteady three-dimensional thermo-hydrodynamic structures in a vertical closed cylinder heated from the side and cooled from above. These authors showed that an increase in temperature difference leads

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Nomenclature

Bi = hL_r/k_1 Biot number
$Fo = \alpha_1/\sqrt{g\beta(T_{hs}-T_0)L_r^3}$ Fourier number
g acceleration of gravity
h heat transfer factor
k_1 thermal conductivity of solid walls
k_2 fluid thermal conductivity
$k_{i,j} = k_i/k_j$ thermal conductivity ratio
l_w solid wall thickness
L_r cylinder radius
p pressure
Pr = ν/α_2 Prandtl number
r radial cylindrical coordinate
R dimensionless radial cylindrical coordinate
$\mathrm{Ra} = g\beta(T_{hs}-T_0)L_r^3/\nu\alpha_2$ Rayleigh number
t time
T temperature
T_0 initial temperature
T_{hs} heat source temperature
V_r velocity along the r-axis
V_φ velocity along the φ-axis
V_z velocity along the z-axis
U dimensionless velocity along the r-axis
V dimensionless velocity along the φ-axis
$V_b = \sqrt{g\beta(T_{hs}-T_0)L_r}$ buoyancy velocity
W dimensionless velocity along the z-axis
z vertical cylindrical coordinate
z_{hs} heat source thickness
Z dimensionless vertical cylindrical coordinate
$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}$ Laplacian
$\tilde{\nabla}^2 = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial Z^2}$ dimensionless Laplacian

Greek symbols
α_1 thermal diffusivity of solid walls
α_2 fluid thermal diffusivity
$\alpha_{i,j} = \alpha_i/\alpha_j$ thermal diffusivity ratio
β coefficient of volumetric thermal expansion
γ inclination angle
$\Delta\tau$ computational time step
Δr computational step along radial cylindrical coordinate
$\Delta\varphi$ computational step along azimuthal cylindrical coordinate
Δz computational step along vertical cylindrical coordinate
Θ dimensionless temperature
Θ^e environmental dimensionless temperature
ν kinematic viscosity
ρ_2 fluid density
τ dimensionless time
φ azimuthal cylindrical coordinate
$\psi_r, \psi_\varphi, \psi_z$ vector potential components
$\Psi_r, \Psi_\varphi, \Psi_z$ dimensionless vector potential components
$\omega_r, \omega_\varphi, \omega_z$ modified vorticity functions
$\Omega_r, \Omega_\varphi, \Omega_z$ dimensionless modified vorticity functions

Subscripts
i, j numbers of the solution domain elements (Fig. 1)
avg average
e environment
hs heat source

to the formation of unstable heat transfer modes. Specifically, at Ra = 2000 the resulting flow is steady but asymmetric. As Ra reaches 3000 the flow already becomes time periodic and oscillates in a large amplitude. Leong [25] in variables such as the vector potential components and the vorticity vector has numerically solved the three-dimensional Rayleigh–Benard convection equations for a vertical cylinder with infinitely thin walls. He has shown three hydrodynamic patterns in the cylinder depending on the Rayleigh number. Cheng et al. [26] investigated influence of the thermal boundary conditions on the three-dimensional convective flow in a vertical cylinder heated from below. These authors revealed that in case of the adiabatic side surface the flow is highly asymmetric and contains multicellular vortices even at symmetric mathematical statement of the problem.

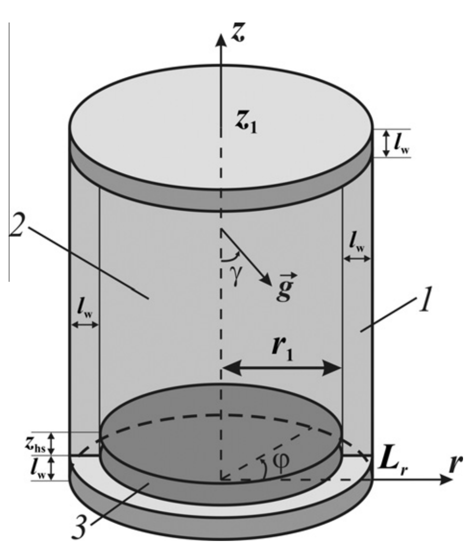

Fig. 1. A scheme of the system: (1) walls; (2) fluid; and (3) heat source.

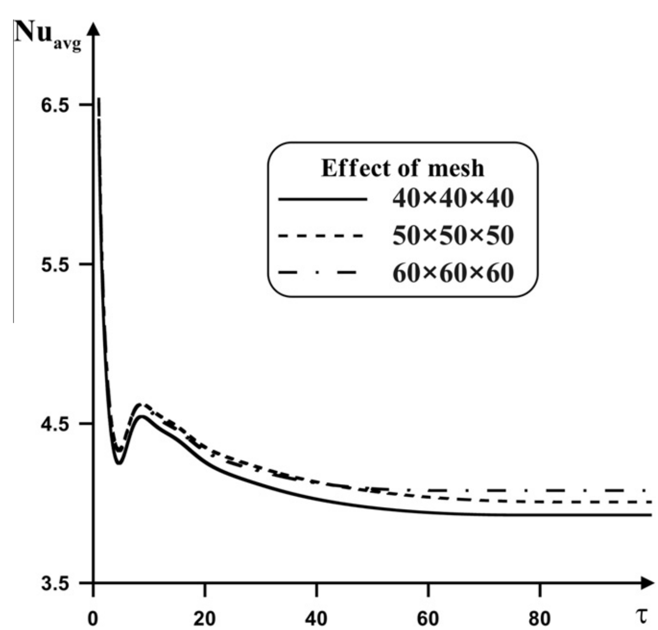

Fig. 2. Variation of the average Nusselt number versus the dimensionless time and the mesh parameters at Ra = 10^4, Pr = 0.7, $k_{2,1} = 5.7 \cdot 10^{-4}$, $\gamma = \pi/6$.