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Onset of Marangoni convection and multiple solutions in a power-law fluid layer under a zero gravity environment

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ABSTRACT

The Marangoni flows in a shallow cavity subject to uniform heat fluxes on all sides are investigated. A power law model is used to characterize the non-Newtonian fluid behavior of the fluid. The system with an underformable free upper surface is assumed to be under a zero gravity environment. The governing parameters for the problem are the thermal Marangoni number *Ma*, power-law index *n*, Prandtl number Pr and cavity aspect ratio aspect ratio *A*. An analytical solution, valid for an infinite layer ($A \gg 1$), is derived on the basis of the parallel flow approximation. For the case of a layer heated from the bottom it is demonstrated that, for shear-thinning fluids ($n < 1$), the onset of convection is subcritical. For shear thickening fluids ($n > 1$), convection is found to occur at a supercritical Rayleigh equal to zero. For the case of a layer heated from all sides it is shown that multiple steady state solutions are possible, some of which are unstable. The effects of the non-Newtonian behavior on the fluid flow, temperature field and heat transfer are discussed. A good agreement is found between the analytical predictions and the numerical results obtained by solving the full governing equations.

1. Introduction

The study of surface tension-driven convection in a layer of fluid is of importance in various areas in engineering and technology. Applications include small-scale hydrodynamics [1] and low gravity hydrodynamics [2,3]. Examples include for instance the growth of crystal and processing materials. The quality of these latter is considerably affected by the strength of the convective motions such that it is of importance to understand and control Marangoni induced convection.

The first study concerning thermocapillary flows in a pure fluid layer, heated from below by a constant temperature or a constant heat flux, is due to Pearson [4]. Based on the linear stability theory this author reported the critical Marangoni number for a layer with non-deformable top surface. Nield [5] also relied on the linear stability theory to investigate the combined effects of surface tension and buoyancy. Extension of Nield's work has been conducted by numerous workers (see for instance Scanlon and Segel [6], Davis [7] and Cloot and Lebon [8]). The role of the deformation of the free surface on the onset of convection has also been investigated by Scriven and Sternling [9], Davis and Homsy [10] and Takashima [11]. It is demonstrated that the existence of the deformable interface can lead to stabilization relative to the case of a planar interface. Hashim and Wilson [12] used a combination of analytical and numerical techniques to predict both steady and overstable convection. Their results, based on the linear stability theory, were obtained for the particular case when the Rayleigh and Marangoni numbers are linearly dependent. The importance of temperature-variable viscosity on the onset of stationary Marangoni convection has been discussed by Awang Kechil and Hashim [13]. It is found that the stability thresholds are critically dependent on large viscosity variations. Three dimensional effects have been investigated by Dauby and Lebon [14] and Bergeron et al. [15]. The critical Marangoni number and the convective pattern at the threshold are obtained as a function of the aspect ratio. A few studies have also been conducted in the case of binary mixture for which surface tension depends on both temperature and concentration (see for instance Bahloul et al. [16]). It was demonstrated that, for this situation, the onset of motion can be subcritical. Crystal growth in microgravity environment was studied experimentally by Croll et al. [17]. It was observed that if the intensity of the flow exceeds a certain level it can become oscillatory and three-dimensional. Kawamura et al. [18] reported a series of microgravity experiments on the Marangoni convection in liquid bridges. The results include the critical temperature differences for the onset of oscillatory flows. The state of the art, on Marangoni convection in microgravity condition, has been summarized in a recent book by Lappa [19].

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Nomenclature

A	aspect ratio of the enclosure, $A = L'/H'$
a	constant
b	constant
C	dimensionless temperature gradient in x-direction
g	gravitational acceleration, m/s^2
H'	height of enclosure, m
k	thermal conductivity, W/(m K)
K	consistency index for a power-law fluid, Pa s^n
L'	width of the enclosure, m
Ma	Marangoni number, $(-\partial\sigma/\partial T')\Delta T' H'^{2n+1}/K\alpha^n$
Ma_C^{sub}	subcritical Marangoni number
Ma_C^{sup}	supercritical Marangoni number
n	power-law index
Nu	Nusselt number, Eq. (11)
Pr	Prandtl number, $(K/\rho\alpha)(\alpha/H'^2)^{n-1}$
q'	constant heat flux per unit area, W/m^2
T	dimensionless temperature
$\Delta T'$	characteristic temperature difference, $q'H'/k$
u	dimensionless velocity x-component
v	dimensionless velocity y-component
x, y	cartesian coordinates measured from the center of the bottom wall of the cavity

Greek Symbols

α	fluid thermal diffusivity, m^2/s
γ	thermal surface tension gradient, K^{-1}
Ψ	dimensionless stream function, Ψ'/α
ρ	density of fluid, kg/m^3
σ	fluid surface tension coefficient, N/m
Ω	dimensionless vorticity, $\Omega'/(\alpha/H'^2)$
μ_a	dimensionless apparent viscosity, $\mu_a = \mu'_a/(\alpha/H'^2)^{n-1}$

Superscript

$'$	dimensional quantities

Subscript

o	refers to the value taken at the centre of the cavity
c	refers to critical conditions

All the above studies are concerned with the case of Marangoni driven convection in a Newtonian fluid layer. However, in many practical applications, the fluid is characterized by a complex rheological behavior such that a Newtonian assumption is inappropriate in practice. Examples include petroleum drilling, chemical reactor design, polymer engineering, geophysical systems, certain separation processes, etc. A few investigations have been undertaken in the past, on natural convection in porous media saturated by non-Newtonian fluids (see for instance Jaluria [20]). However, studies focusing on natural convection in a cavity filled with a non-Newtonian fluid are only a few. Most of the available papers on this topic are concerned with natural convection of a non-Newtonian fluid confined in a cavity differentially heated from the vertical walls. A recent review of the literature for this flow configuration is given by Turan et al. [21,22]. The onset of motion, in a non-Newtonian fluid layer heated from below, was first considered analytically and experimentally by Tien et al. [23] and numerically by Ozoe and Churchill [24]. More recently, a comprehensive review of the literature concerning the Rayleigh-Bénard instability of a non-Newtonian fluid between heated parallel plates is given by Zhang et al. [25]. Buoyant Marangoni convection, in power-law fluid layers subjected to a horizontal constant temperature gradient, has been investigated by Naimi et al [26,27]. The effect of non-Newtonian fluid behavior on the flow pattern, temperature field and heat transfer is discussed. In a subsequent paper these authors [28] investigated analytically and numerically Marangoni convection of a power-law fluid in a zero gravity environment. The effect of Marangoni convection on the flow and heat transfer within a power-law liquid film over a stretching surface has been considered by Chen [29]. It was reported that the velocity and temperature distributions in the film are considerably affected the Marangoni effects. More recently, Zhang et al. [30] considered the problem of thermal Marangoni convection flow of power-law fluid with linear temperature distribution. The effects of power-law index and Marangoni number on velocity and temperature profiles are examined.

The present study focuses on Marangoni induced convection in a shallow cavity, under a zero gravity environment, filled with a non-Newtonian binary fluid. The four faces of the enclosure are exposed to uniform heat fluxes. The power-law model is adopted to characterize the non-Newtonian fluid behavior. In particular, one of the objectives of this paper is to predict the effect of the power-law index n on the onset of Marangoni motion when the layer is heated from the bottom. The paper is organized as follows. In the next sections, the formulation of the problem is presented. The numerical method used to solve the problem is discussed. Then, the parallel flow theory is used to predict the critical Marangoni number for the onset of motion from the rest state. Also, the existence of multiple solutions when the system is subject to cross fluxes is investigated. The last section contains some concluding remarks. The analytical and numerical results presented here are relevant to a better understanding of natural convection in shallow cavity filled with a non-Newtonian fluid.

2. Mathematical formulation of the problem

The physical system consists in a shallow cavity filled with a non-Newtonian fluid of power-law behavior. The enclosure is of height H' and width L'. The origin of the coordinate system is located at the center of the bottom wall of the cavity. Neumann boundary conditions are applied for temperature on the four faces of the system (see Eqs. 7, 8, 9a). The upper surface of the layer is free while the other boundaries are hydrodynamically impermeable. The upper free surface is assumed to be flat and subject to a surface tension σ, which varies with temperature T' as $\sigma = \sigma_0[1 - \gamma(T' - T'_0)]$, where γ is the thermal surface tension gradient. The system is assumed to be under a zero gravity environment.

The dimensionless governing equations describing conservation of momentum and energy are given as:

$$\frac{D\Omega}{Dt} = \Pr\left[\mu_a \nabla^2\Omega + 2\left(\frac{\partial\mu_a}{\partial x}\frac{\partial\Omega}{\partial x} + \frac{\partial\mu_a}{\partial y}\frac{\partial\Omega}{\partial y}\right)\right] + S_\Omega \quad (1)$$

$$\frac{DT}{Dt} = \nabla^2 T \quad (2)$$

where the vorticity Ω, in term of stream function Ψ, is defined as:

$$\nabla^2\Psi = -\Omega \quad (3)$$