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Constructal design of X-shaped conductive pathways for cooling a heat-generating body

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ABSTRACT

This paper applies constructal design to discover the configuration that facilitates the access of the heat that flows through X-shaped pathways of high-conductivity material embedded within a square-shaped heat-generating medium of low-conductivity to cooling this finite-size volume. The objective is to minimize the maximal excess of temperature of the whole system, i.e. the hot spots, independent of where they are located. The total volume and the volume of the material of high thermal conductivity are fixed, but the lengths can vary. It was found numerically that the performance of X-shaped pathways is approximately the same as that the one calculated for a simpler I-shaped blade (i.e. a single pathway of high thermal conductivity material beginning in the heat sink and ending of such a way that there is spacing between the tip of the pathway and the insulated wall) for small values of the high thermal conductivity material and area fraction. However, the X-shaped conductive pathway configuration performs approximately 51% better when compared to I-shaped pathway configuration for larger values of the high-conductivity material and area fraction.

1. Introduction

The problem of cooling with minimal maximal excess of temperature a finite volume that generates heat at every point has been largely studied in the literature [1–16]. Particularly, conductive heat transfer is a very effective way to cooling electronic devices. Bejan and Lorente [17] have shown that when length scales drop a certain level the heat transfer by conduction is the winner to cooling solid bodies when compared to convective heat transfer. The reason is that the available space should be occupied by material that contributes to the purpose of the cooling system [18] and not by ducts that channel the fluid.

Constructal theory is the view that the generation of flow configurations is a physics phenomenon that can be based on a physics principle (the constructal law). The constructal law states that for a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that it provides easier access to the currents that flow through it [1,17,19,20].

Constructal theory has been used to explain deterministically how configurations in nature have been naturally generated, from inanimate rivers to animate designs, such as vascular tissues, locomotion, and social organization [1,17,19,20]. Chief examples are vascular tree-shaped flow architectures, which serve as basis for many rules of animal design [21,22] and river basin design [19,23].

Constructal design is the method based on constructal law to discover the configurations that facilitate the access of the flow currents. The method applies the objective and constraints principle in such a way that the best architecture can emerge deterministically. The applicability of this method to engineered flow systems has been widely discussed in recent literature, e.g. in designing cavities and assembly of fins [24,25]. Constructal design has also been applied successfully to the cooling of electronics by conductive heat transfer on different thermal tree constructs [26–34]. Important contribution to constructal theory literature and constructal heat-generating body problems can be found in Refs. [35–47].

According to constructal theory [17] a fundamental feature of constructal designs for volume-point and area-point paths is that the smallest volume (or area) scale of the flow structure is known and fixed and it is often dictated by, e.g. manufacturing constraints, among others. In this paper we assume that the square elemental volume which generates heat uniformly per unit of volume is cooled by a heat sink at temperature T_0 that is located in the rim. The objective is to minimize the maximal excess of temperature $T_{max} - T_0$. In this sense, X-shaped pathways of higher thermal conductivity instead of a single path is inserted [18,26,27]. For the sake of simplicity, the assumption of two-dimensional problem with the

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Nomenclature

A	area (m^2)
D	thickness (m)
k	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
L	length (m)
q	heat current (W)
q'''	heat uniformly at volumetric rate ($W\,m^{-3}$)
T	temperature (K)
V	volume (m^3)
x, y	coordinates (m)
W	width (m)

Greek symbols

θ	dimensionless temperature, Eq. (7)
ϕ	area fraction

Subscripts

max	maximal
min	single optimization
mm	double optimization
opt	optimal
oo	twice optimized
p	path (blades) of high thermal conductivity
0	isothermal wall, single blade
1	X-blades

Superscript

(~)	dimensionless variables, Eqs. (5)–(9), (11)–(13), (15)

thermal conductivity ratio and the volume (area) fraction being two design parameters of the conductive composite is made.

2. Mathematical model

Consider the conducting body shown in Fig. 1. The configuration is two-dimensional, with the third dimension (W) sufficiently long in comparison with the length L of the total volume. There are X-shaped pathways of high thermal conductivity k_p material embedded in the body of lower thermal conductivity k. The solid body generates heat uniformly at the volumetric rate q''' (W/m^3). The outer surfaces of the solid are perfectly insulated. The generated heat current ($q'''AW$) is removed by the heat sink located in the rim of the body at temperature T_0.

The work consists to calculate the dimensionless maximal excess of temperature $(T_{max} - T_0)/(q'''A/k)$ and see what geometry (L_1/L_0, D_1/D_0) facilitates the most the heat flow removal. According to constructal design, this search can be subjected to two constraints, namely, the total area constraint,

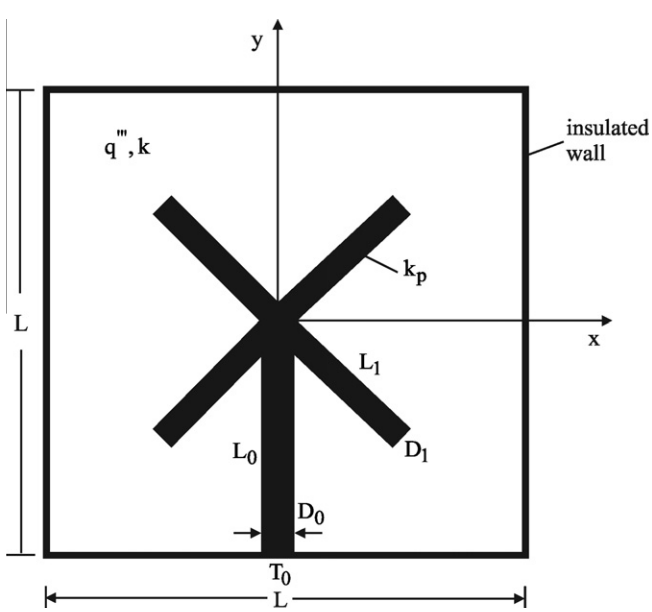

Fig. 1. Body of low conductivity and heat generation with X-shaped blades of higher conductivity.

$$A = L^2 \tag{1}$$

and the area occupied by the high conductivity material,

$$A_p = 4D_1L_1 + D_1^2 + D_0L_0 + \frac{D_0^2}{4} \tag{2}$$

Eqs. (1) and (2) can be expressed as the area fraction

$$\phi = \frac{A_p}{A} \tag{3}$$

Due to pure observation it was noted that another geometric constraint emerged and it was given by

$$\frac{L}{2} = L_0 + \frac{D_0}{2} + \cos(\pi/4)D_1 \tag{4}$$

The analysis that delivers the maximal excess of temperature as a function of the geometry consists of solving numerically the steady heat conduction with heat generation equation along the lower conductivity k-region,

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} + 1 = 0 \tag{5}$$

and the steady heat conduction without heat generation equation in the k_p-region

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} = 0 \tag{6}$$

where the dimensionless variables are given by

$$\theta = \frac{T - T_0}{q'''A/k}, \quad \tilde{k}_p = k_p/k \tag{7}$$

and

$$\tilde{x}, \tilde{y}, \tilde{L}, \tilde{L}_0, \tilde{L}_1, \tilde{D}_0, \tilde{D}_1 = \frac{x, y, L, L_0, L_1, D_0, D_1}{L} \tag{8}$$

The outer surfaces are insulated and their boundary conditions are

$$\frac{\partial\theta}{\partial\tilde{n}} = 0 \tag{9}$$

The boundary condition in the range $(-\tilde{D}_0/2 \leqslant \tilde{x} \leqslant \tilde{D}_0/2; \tilde{y} = -\tilde{L}/2)$ which is in contact with the heat sink is given by

$$\theta_0 = 0 \tag{10}$$

The dimensionless form of Eqs. (1), (3) and (4) are

$$1 = \tilde{L}^2 \tag{11}$$