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Magnetogasdynamic flow and heat transfer in a microchannel with isothermally heated walls

Huei Chu Weng *, Dave C. Chen [1]

Department of Mechanical Engineering, Chung Yuan Christian University, Chungli 32023, Taiwan, ROC

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ABSTRACT

The presence of ionized gas flow in an applied electric and magnetic field could result in an electromagnetic driving force and a joule heating. It is desirable to understand the causes of the two mechanisms and their roles on ionized gas microflow and heat transfer. In this study, a mathematical model is developed of the pressure-driven gas flow through a long isothermally heated horizontal planar microchannel under an applied electric and magnetic field. The fully developed solutions of the flow and thermal field distributions as well as the corresponding characteristics are derived analytically and presented in terms of dimensionless parameters. It is found that an electromagnetic driving force can be produced by a combined non-zero electric field and negative magnetic field and results in an additional velocity slip and an additional flow drag. Also, a joule heating can be produced only by an electric field and enhanced by a positive magnetic field and results in an additional temperature jump and an additional heat transfer. The effects of electromagnetic driving force and joule heating on velocity slip and temperature jump can be magnified by increasing the Knudsen number; however, the force and heating effects on flow drag and heat transfer rate are found to be diminished.

1. Introduction

Recent advances in science and technology have promoted a rapid development of different microfluidic devices in physical, chemical, biological, medical, engineering, and energy-related fields. Due to the difficulty in making precise measurements, microscale gas flow and heat transfer modeling was emphasized in modern microfluidic applications, such as microelectrochemical cell transport, microheat exchanging, and microchip cooling, etc.

The small dimensions encountered in microfluidic devices can result in gas rarefaction. The rarefied gas flow in these devices was divided into different regimes based on a dimensionless quantity called the Knudsen number Kn, defined as the ratio of the molecular mean free path to the characteristic length [1]. At standard atmospheric conditions, most microfluidic devices operate in the entire slip regime $0.01 < \mathrm{Kn} \leqslant 0.1$ [2]. Solving NS-based hydrodynamic equations subject to slip boundary conditions is one of the feasible methods to model slip flow (see for example [3–7]). Numerous theoretical investigations have then been carried out on microscale convection for slip flow in the past decade. For forced convection, Tunc and Bayazitoglu [8] performed an analytical study of fully developed convection in an isoflux rectangular microchannel by solving the Navier–Stokes and energy equations subject to the first-order Maxwell slip and local heat flux boundary conditions. Renksizbulut et al. [9] investigated the developing convection in an isothermal rectangular microchannel by using the first-order Maxwell slip and Smoluchowski jump boundary conditions. Shojaeian and Dibaji [10] numerically obtained a fully developed solution for an isothermal triangular microchannel. Sadeghi and Saidi [11] placed emphasis on the importance of viscous dissipation in fully developed forced convection by considering planar and annular microchannels with asymmetric wall heat fluxes. As for natural convection, Chen and Weng [12] analytically studied the first-order fully developed convection in a vertical planar microchannel with asymmetric wall temperatures. Haddad et al. [13] reported implicit finite-difference simulations of the developing convection in an isothermal planar microchannel filled with porous media. Biswal et al. [14] numerically investigated the flow and heat transfer characteristics in the developing region of an isothermal planar microchannel by using the semi-implicit method for pressure linked equations. Chen and Weng [15] modeled the developing convection in an asymmetrically heated planar microchannel based on the second-order Beskok-Karniadakis slip/jump boundary conditions by using a marching implicit procedure. They found that the second-order conditions play an important role in the developing region. Chakraborty et al. [16] presented

* Corresponding author. Address: Room 513 (5F), Engineering Building. Tel.: +886 3 2654311; mobile: +886 912704099; fax: +886 3 2654399.

E-mail addresses: hcweng@cycu.edu.tw (H.C. Weng), tp6fup4@yahoo.com.tw (D.C. Chen).

URL: http://home.educities.edu.tw/hcweng/ (H.C. Weng).

[1] Tel.: +886 3 2654331.

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a boundary layer integral analysis to investigate the heat transfer characteristics of the first-order developing convection in an isothermal planar microchannel. Weng and Chen [17,18] examined the roles of variable physical properties and wall-surface curvature in first-order fully developed convection with asymmetric heating. Weng and Chen [19] performed an analytical study of first-order fully developed convection in a vertical planar microchannel with asymmetric wall heat fluxes. Buonomo and Manca [20] further reported implicit finite-difference simulations of the developing convection. For mixed convection, Avci and Aydin [21] analytically investigated the first-order fully developed mixed convection in a vertical planar microchannel with asymmetric wall temperatures. Avci and Aydin [22,23] further extended their study to the case where isoflux walls are considered and the case where an annular cross-section, formed by two concentric microtubes, is considered. Weng and Jian [24] numerically examined the developing convection in an isothermal planar microchannel on the basis of second-order Maxwell–Smoluchowski–Burnett slip/jump boundary conditions. Jian and Weng [25] analytically investigated the effect of second-order slip on the mixed convection through a long heated vertical planar microchannel with asymmetric wall temperatures.

Since the flow and heat transfer of gases at the microscale could easily be influenced by an external electric and magnetic field, magnetogasdynamics (MGD) promises to be useful for electromagnetically controllable microfluidic devices. Recently, Cai and Liu [26] initiated the analytical NS-based first-order study of the MGD quasi-isothermal flow in a finite horizontal planar microchannel with a low-magnetic-Reynolds-number assumption. They found that the effects of electromagnetic force and joule heating on the flow properties could be significant. Shojaeian and Shojaeian [27] further analytically studied the flow and heat transfer in a long planar microchannel with constant wall heat fluxes. The fully developed results were found that on increasing the magnetic field strength, both the flow drag and the heat transfer rate are enhanced, whereas it tends to suppress the flow. Further, it was found that the magnetic effect is minified by increasing the Knudsen number. Until recently however, there were no studies that specifically examine how an electric and magnetic field produces an electromagnetic driving force and a joule heating and how the two mechanisms affect the microscale flow and heat transfer with constant wall temperatures. In this study, to provide a more detailed microscale theory of MGD, the hydrodynamic and electromagnetic equations as well as the slip and jump boundary conditions are summarized first. Then, for simplicity of analysis, a mathematical model of pressure-driven incompressible gas flow through a long isothermally heated horizontal planar microchannel in the presence of an applied electric and magnetic field is developed. The fully developed solutions for flow and thermal field distributions as well as the corresponding characteristics are derived analytically and presented in terms of dimensionless parameters, so as to further investigate the causes of the two mechanisms and their roles on ionized gas microflow and heat transfer.

2. Field equations

In this section, we introduce the fundamental equations on which microscale magnetogasdynamic flow and heat transfer is based.

2.1. Hydrodynamic and electromagnetic equations

The conservation of mass is the key to tracking flowing fluid. The statement for a control volume is

$$\frac{\partial M_{cv}}{\partial t} = \sum_{\dot{m}_{in}} - \sum_{\dot{m}_{out}}. \tag{2.1}$$

The conservation of linear momentum leads to the conclusion that a force can result from or cause a change in fluid velocity (magnitude and/or direction), and the conservation of energy leads to the conclusion that a work and/or a heat can result from or cause a change in fluid velocity and/or a change in fluid temperature. Their statement for a control volume are

$$\frac{\partial (Mu_n)_{cv}}{\partial t} = \sum \dot{m}u_n)_+ - \sum \dot{m}u_n)_- + \sum F_n)_+ - \sum F_n)_-, \tag{2.2}$$

$$\frac{\partial (Me)_{cv}}{\partial t} = \sum \dot{m}e)_{in} - \sum \dot{m}e)_{out} + \sum \dot{Q}_{in} - \sum \dot{Q}_{out} + \sum \dot{W} + \dot{O}. \tag{2.3}$$

Here, t is the time, $\partial/\partial t$ is the time rate of change in a laboratory frame of reference, M is the instantaneous mass, $\dot{m}$ is the mass flow rate, u is the velocity, e is the specific internal energy, F is the force acting on the control volume, $\dot{Q}$ is the energy change rate associated with heat conduction, $\dot{W}$ is the energy change rate associated with force, $\dot{O}$ is the internal heat generation rate, the subscript cv denotes the values in the control volume, the subscript n denotes the coordinate chosen for analysis, the subscripts *in* and *out* indicate the values for flow into and out of the control volume, respectively, and the subscripts + and − denote the values in the positive and negative directions, respectively.

Following the infinitesimal control volume approach, Eqs. (2.1)–(2.3) require that

$$\frac{d\rho}{dt} + \rho\nabla\cdot\mathbf{u} = 0, \tag{2.4}$$

$$\rho\frac{d\mathbf{u}}{dt} = \nabla\cdot\mathbf{T} + \mathbf{f}, \tag{2.5}$$

$$\rho\frac{de}{dt} = -\nabla\cdot\mathbf{q} + \mathbf{T}^S : \mathbf{D} + o, \tag{2.6}$$

where d/dt is the material derivative, $\mathbf{u}$ is the velocity vector, $\mathbf{T}$ is the stress tensor, given by

$$\mathbf{T} = (-p + \eta\nabla\cdot\mathbf{u})\mathbf{I} + 2\mu\mathbf{D}, \tag{2.7}$$

$\mathbf{f}$ is the body force vector per unit volume and can be divided into a gravity force vector $\mathbf{f}_g$ and an electromagnetic force vector $\mathbf{f}_{em}$, given by

$$\left.\begin{aligned} \mathbf{f}_g &= \rho\mathbf{g}, \\ \mathbf{f}_{em} &= \mathbf{j}\times\mathbf{b} = \sigma(\mathbf{e} + \mathbf{u}\times\mathbf{b})\times\mathbf{b}, \end{aligned}\right\} \tag{2.8}$$

$\mathbf{q}$ is the heat flux vector and can be related to the temperature field by Fourier's law of conduction, given by

$$\mathbf{q} = -k\nabla T, \tag{2.9}$$

o is the internal heat generation rate per unit volume and can be represented by the joule heating (or ohmic dissipation) o_{em}, given by

$$o_{em} = |\mathbf{e}||\mathbf{j}| = \sigma|\mathbf{e}||\mathbf{e} + \mathbf{u}\times\mathbf{b}|. \tag{2.10}$$

In Eqs. (2.7)–(2.10), $\mathbf{D}$ is the deformation rate tensor, $\mathbf{D} = ((\nabla\mathbf{u})^T + \nabla\mathbf{u})/2$, $\mathbf{I}$ is the Kronecker delta tensor, $\mathbf{g}$ is the gravitational field vector, $\mathbf{j}$ is the electric current density induced by the effective electric field $\mathbf{e} + \mathbf{u}\times\mathbf{b}$ acting on a fluid of electrical conductivity σ, $\mathbf{e}$ is the electric field vector, $\mathbf{b}$ is the magnetic induction field vector, p is the pressure, T is the temperature, ρ is the fluid density, μ is the shear viscosity, η is the bulk viscosity, related to μ by $\eta = -2\mu/3$, k is the thermal conductivity, the superscript T indicates the transpose of a tensor, and the superscript T, S indicate the transpose and symmetric part of a tensor, respectively. Note that the electrical conductivity of a gas depends mainly on ion concentration.