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Fourier method for heat transport equation in the convergent channel

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ABSTRACT

The heat transfer processes for stationary flow of Newtonian viscous fluid between two inclined planes (convergent channel) are considered. The solution is obtained by Fourier method. As the results of research temperature and velocity fields, the expressions for power and average temperature at the outlet are obtained in the form of the series. The problem of fluid flow in the plane slot channel was solved similarly. On the basis of the obtained formulas thermal and hydrodynamic fields for the flow of water in the convergent channel were calculated. Comparison of thermal effectiveness of convergent and plane slot channels was made.

1. Introduction

Solution of the problem in analytical form is the most convenient form for analysis of the problem under consideration and generalization of the results while the realization of most numerical methods is based on knowingly approximate algorithms. These methods lose computational stability and diverge giving wrong solutions due to error accumulation. Classical solutions are supporting and serve as zero approximations for more complex applications. Therefore it is extremely important to know their properties and peculiarities in different ranges of parameter values. Undoubtedly the problem of stationary motion of viscous incompressible fluid between two planes inclined to each other refers to the number of such kind of well-known problems of continuum mechanics. Moreover the study of converging flows in a plane convergent channel is important for technological applications, for example, production of plastics and metal sheets.

Today a large number of works is devoted to the study of heat transfer problem when fluid is flowing in convergent and divergent channels. Some of them are experimental. Experimental study of natural convection of air in a vertical convergent channel with uniformly heated plane walls is presented in [1]. Flow visualization is obtained numerically as well as experimentally. Rather good agreement of numerical and experimental data is revealed. The results are presented in the form of the wall temperature profiles as a function of convergent angel, distance between the walls and heat flux. Experimental studies of heat transfer for flow in a vertical convergent channel one wall of which is vertical uniformly heated and the opposite wall has a convergent channel of 3° and is isolated are carried out in [2]. Formation of hydrodynamic and thermal turbulent boundary layers which increase the velocity in the direction of flow is studied in [3]. Spatial motion of non-Newtonian fluid when lean solution of polyethylene oxide is flowing through the tiny convergent channel is studied in [4]. Pressure drop and flow velocity in the channel were measured and flow field was obtained. A lot of papers are devoted to numerical study. Forced convection of compressible mediums flowing in three-dimensional conical chimney is numerically studied in [5]. Roe schemes were used to solve the problem of compressible flow at low velocities. Computer calculations were performed on the computational platform CUDA. Work [6] presents the method for obtaining temperature charts for natural convection in air in the vertical convergent channel. Optimal distance and convergent angle are obtained in terms of thermal effectiveness based on correlation of mean Nusselt number and dimensionless wall temperature as a function of Rayleigh number. Heat transfer by natural convection in a vertical convergent channel with constant wall temperature and using air as a working medium is numerically studied in [7]. Characteristics of heat transfer and pressure drop in the convergent and divergent channels of rectangular cross-section based on the modeling of flow using three-dimensional parabolic model are obtained in [8].

Quiet enough attention was given to analytical solution of heat exchange problems when fluids are flowing in various channels. The Graetz's works (1883) should be emphasized among this kind of works. He used the variable separation method to obtain temperature distribution in a circular pipe for the two cases: parabolic and uniform velocity profiles. Solutions obtained by Graetz were

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Nomenclature

a	thermal diffusivity of the fluid (m^2/s)
h	plane slot channel half width (m)
L	plane slot channel length (m)
$\vec{n}$	normal to a channel wall
N	power required to move a fluid through the convergent channel (W/m)
N_0	power required to move a fluid through the plane slot channel (W/m)
p	pressure (Pa)
Q	flow rate (m^2/s)
Q_w	quantity of heat transferred through the convergent channel wall (W/m)
Q_{w0}	quantity of heat transferred through the plane slot channel wall (W/m)
q_w	heat-flux density on the channel boundary (W/m^2)
r, φ, z	cylindrical coordinates
r_1, r_0	radial coordinates at the convergent channel inlet and outlet respectively (m)
T	temperature (K)
T_0	inlet temperature (K)
T_w	wall temperature (K)
v	longitudinal velocity component in the plane slot channel (m/s)
$v_{\max}$	maximum velocity on the axis of convergent channel (m/s)
v_r	radial component of a velocity vector (m/s)
x	dimensionless variable
x_1, y_1	Cartesian coordinates (m)

Greek symbols

α	a half convergent channel angle (rad)
λ_f	thermal conductivity of the fluid (W/mK)
λ_p	eigenvalues of the Sturm–Liouville problem
μ	dynamic viscosity (Pa s)
θ	dimensionless temperature
θ_s	average temperature on the exit (K)
ρ	density (kg/m^3)
ψ	dimensionless angle
ψ_p	eigenfunctions of the Sturm–Liouville problem

developed by Sellars, Tribus and Klein (1956), Drew (1931). However the way to obtain the solution for the convergent and divergent channels using the analytical method was not described in the literature therefore the present work is devoted to the solution of this problem.

2. Solution of momentum transport equation

We define the velocity field when viscous Newtonian fluid is stationary flowing in the channel formed by two plane walls inclined to each other suggesting that the variation range of the temperature and fluid properties are such that a change of its heat-transfer properties with temperature can be neglected. We assume that gravity is negligible small, velocity has only radial component v_r and fluid sticks on the walls of the channel. In scientific literature this one-dimensional flow is widely known as the solution of Jeffery–Hamel problem [9].

Fig. 1 shows the convergent channel. Two planes restricting the flow intersect at an angle of 2α. Cylindrical coordinate system is used. Z axis coincides with the intersection line of these planes. Fluid flows in the direction shown by the arrow.

Now we obtain the velocity field. Under the assumptions made the system of equations of motion and continuity takes the form

$$\rho v_r \frac{\partial v_r}{\partial r} = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \varphi^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{v_r}{r^2}\right), \quad (1)$$

$$\frac{1}{r}\frac{\partial p}{\partial \varphi} = \mu\left(\frac{2}{r^2}\frac{\partial v_r}{\partial \varphi}\right), \quad (2)$$

$$\frac{\partial r v_r}{\partial r} = 0. \quad (3)$$

Here the equation in projection on the z axis is performed automatically.

Boundary conditions of adhesion of the fluid on the walls can be written as $v_r(r, \pm\alpha) = 0$

In addition we will consider that flow rate through the channel cross-section is given as $Q = \int_{-\alpha}^{\alpha} r v_r d\varphi$ here $Q < 0$ for convergent channels.

It follows from Eq. (3) that

$$r v_r = f(\varphi). \quad (4)$$

Substituting (4) in (1) and (2) and then making some transformations we get

$$\frac{\rho}{\mu}\frac{d}{d\varphi}(f^2(\varphi)) + \frac{d^3 f(\varphi)}{d\varphi^3} + 4\frac{df(\varphi)}{d\varphi} = 0. \quad (5)$$

Integrating (5) once with respect to φ we obtain

$$\frac{\rho}{\mu} f^2(\varphi) + \frac{d^2 f(\varphi)}{d\varphi^2} + 4f(\varphi) + C_0 = 0, \quad (6)$$

where C_0 = *const*

We introduce the dimensionless angle $\psi = \frac{\varphi}{\alpha}$ and a new function

$$g(\psi) = \frac{\rho \cdot f(\varphi)}{\mu \cdot u_0}, \quad (7)$$

where $u_0 = \frac{\rho}{\mu} f(0)$, then (6) can be written as

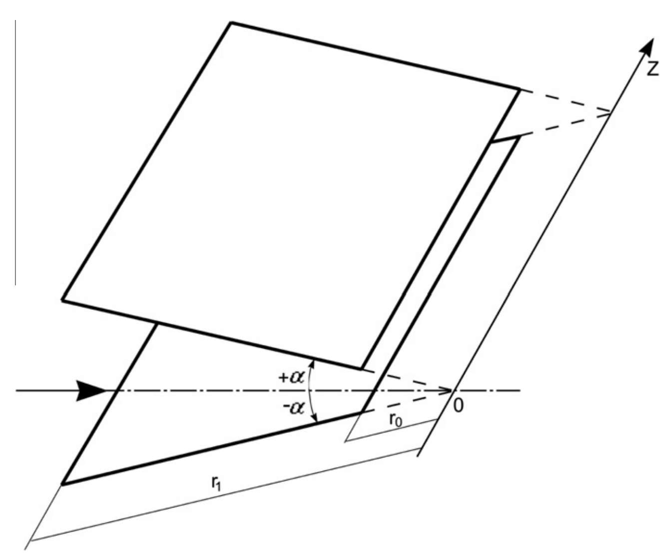

Fig. 1. Scheme of the convergent channel.